AQA

A-level
MATHEMATICS
7357/1

Paper 1

Mark scheme

June 2025

Version: 1.0 Final

2 5 6 A 7 3 5 7 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
FT	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
ISW	ignore subsequent working
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student’s best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles third option	1.2	B1	$3e^x$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles first option	2.2a	R1	Convergent
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles second option	1.1b	B1	$\log_3 2$
	Question 3 Total		**1**	

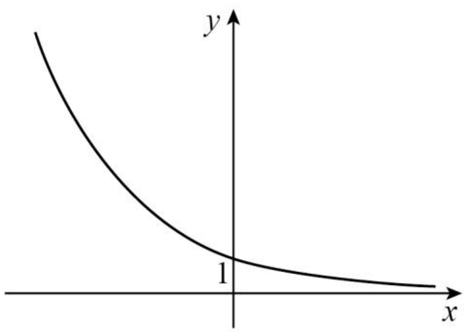
Q	Marking instructions	AO	Marks	Typical solution
4(a)	Circles third option	1.1b	B1	$\|x\| < \frac{1}{8}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Circles second option	1.1b	B1	–4
	Subtotal		**1**	

	Question 4 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5	Integrates at least one term correctly, may be unsimplified	1.1a	M1	$\int\left(4x-3+x^{-\frac{1}{2}}\right)\mathrm{d}x$ $=\dfrac{4x^2}{2}-3x+\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$ $=2x^2-3x+2x^{\frac{1}{2}}+c$
	Integrates at least two terms correctly, may be unsimplified	1.1a	M1	
	Obtains $2x^2-3x+2x^{\frac{1}{2}}+c$ or $2x^2-3x+2\sqrt{x}+c$ or $2x^2-3x+2x^{0.5}+c$	1.1b	A1	
	Question 5 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6	Substitutes $n=250$ in $\dfrac{n}{2}$ and $(n-1)$ and at least one of $a=3$ or $d=-0.5$ into $\dfrac{n}{2}\left(2a+(n-1)d\right)$ Or into $a+(n-1)d$ and $\dfrac{n}{2}(a+l)$ note the correct value of l = -121.5 Or into $\sum_{1}^{250} a+(n-1)d$ Condone poor bracketing	1.1a	M1	$S_{250}=\dfrac{250}{2}\left(2\times3+(250-1)\times-0.5\right)$ $=-14812.5$
	Obtains –14812.5 or $-\dfrac{29625}{2}$	1.1b	A1	
	Question 6 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7	Sketches an exponential graph which does not cross the x-axis. Graph must be in 1st and 2nd quadrants. Condone the correct shape reflected in y-axis. Condone minor errors at the extremes.	1.1a	M1	
	Sketches fully correct graph with y-intercept labelled as 1 or (0,1)	1.1b	A1	
	Question 7 Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8	Replaces sin $5x$ with $5x$ **Or** Replaces tan x with x In $\dfrac{4+\sin 5x-3\cos 2x}{1+2\tan x}$	1.1a	M1	$\dfrac{4+5x-3\left(1-\dfrac{(2x)^2}{2}\right)}{1+2x}$ $=\dfrac{4+5x-3\left(1-2x^2\right)}{1+2x}$ $=\dfrac{6x^2+5x+1}{1+2x}$ $=\dfrac{(2x+1)(3x+1)}{1+2x}$ $=3x+1$ $\therefore y=3x+1$
	Obtains $3\left(1-\dfrac{(2x)^2}{2}\right)$ Condone $3\left(1-\dfrac{2x^2}{2}\right)$	1.1a	M1	
	Obtains $\dfrac{4+5x-3\left(1-\dfrac{(2x)^2}{2}\right)}{1+2x}$ OE	1.1b	A1	
	Uses a valid method to simplify their fraction to a linear form. May see evidence of factorising, algebraic division or comparing coefficients.	3.1a	M1	
	Completes a reasoned argument to obtain $y=3x+1$	2.1	R1	
	Question 8 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Uses $0.2a = 60$ PI by $a = 300$	1.1a	M1	$0.2a = 60$ $a = 300$ $S_5 = \dfrac{300\left(1-0.2^5\right)}{1-0.2}$ $= 374.88$
	Deduces $a = 300$ PI by the use of $\dfrac{60}{0.2}$	2.2a	R1	
	Uses a complete method to find the sum of the five terms. For example Uses $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ with their $a \neq 60$, $r = 0.2$ and $n = 5$ **Or** Uses $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ with $a = 60$, $r = 0.2$ and $n = 4$ and adds their first term **Or** Uses their $a = 300$ and $r = 0.2$ to calculate all five terms and finds their sum 300 + 60 + 12 + 2.4 + 0.48	3.1a	M1	
	Obtains $\dfrac{9372}{25}$ OE	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	States $\left(T_\infty =\right)\dfrac{a}{1-r}$ and $ar = 60$ or $a = \dfrac{60}{r}$ **Or** States $\left(T_\infty =\right)\dfrac{60/r}{1-r}$	3.1a	M1	$ar = 60 \Rightarrow a = \dfrac{60}{r}$ $T_\infty = \dfrac{a}{1-r}$ $= \dfrac{60}{r(1-r)}$ $= \dfrac{60}{r-r^2}$
	Completes a reasoned argument to show $T_\infty = \dfrac{60}{r-r^2}$ Must start from $\left(T_\infty =\right)\dfrac{a}{1-r}$ and $ar = 60$ or $a = \dfrac{60}{r}$ AG	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Deduces $(r =)\dfrac{1}{2}$	2.2a	M1	$r-r^2$ is at its maximum when $r = \dfrac{1}{2}$ Maximum value of $r-r^2 = \dfrac{1}{4}$
	Deduces maximum value of $r-r^2 = \dfrac{1}{4}$	1.1b	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(iii)	Obtains $\dfrac{60}{\text{their } r-r^2}$ provided their $r-r^2 \neq 0$ PI by 240	2.2a	M1	Min $T_\infty = \dfrac{60}{\frac{1}{4}} = 240$ Hence $T_\infty \geq 240$
	Deduces $T_\infty \geq 240$	2.2a	R1	
	Subtotal		**2**	

	Question 9 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Obtains $\ln Ae^{-kt} = \ln(21 \pm \theta)$ or $\ln e^{-kt} = \ln\dfrac{21 \pm \theta}{A}$ or $-kt = \ln\dfrac{21 \pm \theta}{A}$	3.1a	M1	$\theta = 21 - Ae^{-kt}$ $Ae^{-kt} = 21 - \theta$ $\ln Ae^{-kt} = \ln(21 - \theta)$ $\ln A + \ln e^{-kt} = \ln(21 - \theta)$ $\ln A - kt = \ln(21 - \theta)$
	Obtains $\ln Ae^{-kt} = \ln A + \ln e^{-kt}$ or $\ln\dfrac{21 \pm \theta}{A} = \ln(21 \pm \theta) - \ln A$	1.1a	M1	
	Completes reasoned argument to obtain $\ln(21 - \theta) = -kt + \ln A$ AG	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	States $\ln A = 3.676$	3.4	M1	$\ln A = 3.676$ $A = e^{3.676} = 39.5$
	Obtains AWRT 39.5	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Obtains $\pm\dfrac{3.676}{19.98}$ **Or** Substitutes their value of A or $\ln A = 3.676$, $t = 19.98$ and $\ln(21-\theta) = 0$ to form an equation for k	3.4	M1	$-k = -\dfrac{3.676}{19.98}$ $k = 0.1839...$ $= 0.184$
	Obtains AWRT $k = 0.184$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iii)	Uses the model $\theta = 21 - Ae^{-kt}$ with their A or $\ln A = 3.676$ and $t = 0$ **Or** Equates $\ln(21-\theta)$ to 3.676	3.4	M1	$\theta = 21 - 39.5e^{0}$ $= -18.5\,^{\circ}C$
	Obtains AWRT $-18.5\,^{\circ}C$ Condone missing units CAO	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Uses either version of the model with their A from (b)(i) or $\ln A = 3.676$, and their k from their (b)(ii) and $\theta = 4$	3.4	M1	$4 = 21 - 39.5e^{-0.184t}$ $t = 4.581996...$ Time = 4 hrs 30 mins
	Obtains AWRT 4.58 PI by 4 hrs 30 mins (270min) or 4 hrs 40 mins (280 min)	1.1b	A1	
	Obtains 4 hrs 30 mins or 4 hrs 40 mins Accept 270 min or 280 min Answer must not come from an incorrect value of t	3.2a	A1	
	Subtotal		**3**	

	Question 10 Total		**12**	

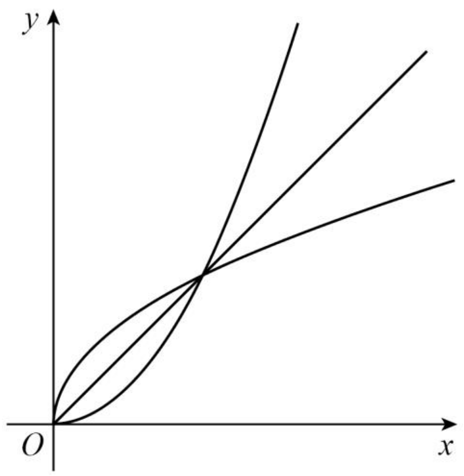
Q	Marking instructions	AO	Marks	Typical solution
11(a)	Uses implicit differentiation with $x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ seen	1.1a	M1	$x^2y+4y^3=8x$ $2xy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}+12y^2\frac{\mathrm{d}y}{\mathrm{d}x}=8$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow 2xy=8$ $\Rightarrow y=\frac{4}{x}$
	Uses the product rule to obtain $Bxy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}$	3.1a	M1	
	Obtains $2xy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}+12y^2\frac{\mathrm{d}y}{\mathrm{d}x}=8$ OE	1.1b	A1	
	Completes a reasoned argument using $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and $2xy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}+12y^2\frac{\mathrm{d}y}{\mathrm{d}x}=8$ OE with at least one correct line of intermediate working before showing that $y=\frac{4}{x}$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Obtains $x^2\times\frac{4}{x}+4\left(\frac{4}{x}\right)^3=8x$ OE	3.1a	B1	$x^2\times\frac{4}{x}+4\left(\frac{4}{x}\right)^3=8x$ $4x+\frac{256}{x^3}=8x$ $x^4=64$ $x=\pm\sqrt{8}$
	Rearranges their equation to obtain $ax^4=k$ or $ax^{-4}=k$ OE	2.1	M1	
	Deduces $x=\pm\sqrt{8}$ CAO	2.2a	R1	
	Subtotal		**3**	

	Question 11 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Deduces ≥ 5 or >5	2.2a	M1	Range $\{y : y \geq 5\}$
	Writes the correct range using correct set notation Acceptable examples include: $\{x : x \geq 5\}$ $\{\mathrm{f}(x) : \mathrm{f}(x) \geq 5\}$ $[5, \infty)$ Ignore anything before $[5, \infty)$	2.5	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Demonstrates that f is many to one. This could be evidenced by a sketch of the graph of y=f(x) demonstrating the horizontal line test, or by giving two x-values which result in the same value of $\mathrm{f}(x)$, or states $\mathrm{f}(-x) = \mathrm{f}(x)$	2.4	E1	$\mathrm{f}(-1) = 6 = \mathrm{f}(1)$ $\therefore$ f is many to one, so it does not have an inverse
	Explains that f is many to one or that f is not one to one and deduces that f does not have an inverse	2.2a	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Draws a convex curve between the origin and (1,1) There should be no doubt that their graph intersects (1,1) Ignore anything outside of quadrant 1	1.1a	M1	
	Sketches a fully correct graph of $y = x^2$ for $x \geq 0$ Their curve must be in quadrant 1 only.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(d)(i)	Obtains $\sqrt{x^2+5}$ No ISW	1.1b	B1	$\sqrt{x^2+5}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(d)(ii)	Deduces the range of h Accept $y \geq \sqrt{5}$, $\mathrm{gf}(x) \geq \sqrt{5}$ Condone $\mathrm{h} \geq \sqrt{5}$, $\mathrm{gf} \geq \sqrt{5}$ Or accept set notation in ACF	2.2a	R1	$\mathrm{h}(x) \geq \sqrt{5}$
	Subtotal		**1**	

	Question 12 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Obtains $32(4t+1)$ Can be unsimplified OE	1.1b	B1	$\frac{dx}{dt}=32(4t+1)$ $\frac{dy}{dt}=-4e^{-4t}$ $\frac{dy}{dx}=\frac{-4e^{-4t}}{32(4t+1)}=\frac{-e^{-4t}}{8(4t+1)}$
	Obtains $-4e^{-4t}$	1.1b	B1	
	Uses their $\frac{dy}{dt}$ ÷ their $\frac{dx}{dt}$ to obtain an expression for $\frac{dy}{dx}$ Can be unsimplified ISW ACF	3.1a	B1F	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Substitutes $t=0$ to obtain values for x, y and their $\frac{dy}{dx}$ PI by correct values	3.1a	M1	$t=0$ $x=4$ $y=1$ $\frac{dy}{dx}=-\frac{1}{8}$ $y-1=-\frac{1}{8}(x-4)$
	Obtains at least two of $x=4$, $y=1$ or $\frac{dy}{dx}=-\frac{1}{8}$	1.1a	M1	
	Obtains $y-1=-\frac{1}{8}(x-4)$ ACF ISW	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	Eliminates t correctly to form a Cartesian equation. May use positive or negative square root or both. Condone a sign error when rearranging either parametric equation.	3.1a	M1	$4t+1=\frac{\pm\sqrt{x}}{2}$ The positive square root is used since $-\frac{1}{4}\le t\le 0$ $4t=\frac{\sqrt{x}-2}{2}$ $y=\mathrm{e}^{-\frac{\sqrt{x}-2}{2}}$
	Obtains $y=\mathrm{e}^{1-\frac{\sqrt{x}}{2}}$ or $y=\mathrm{e}^{1+\frac{\sqrt{x}}{2}}$ or $y=\mathrm{e}^{1\pm\frac{\sqrt{x}}{2}}$ OE May be unsimplified.	1.1a	M1	
	Gives a valid explanation for selecting the appropriate square root for example $-\frac{1}{4}\le t$ or $y\le \mathrm{e}$ **AND** Obtains $y=\mathrm{e}^{1-\frac{\sqrt{x}}{2}}$ OE	2.4	R1	
	Subtotal		**3**	

	Question 13 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
14	Begins integration by parts by writing $u = 4x$ $v' = \sin 2x$ $u' = 4$ $v = A\cos 2x$ PI by $4Ax\cos 2x - 4A\int (\cos 2x)\,dx$ **Or** $-2x\cos 2x + \sin 2x(+c)$ **Or** $u = \sin 2x$ $v' = 4x$ $u' = A\cos 2x$ $v = 2x^2$ PI by $2x^2 \sin 2x - 2A\int (x^2 \cos 2x)\,dx$ In either approach the constant 4 may be contained within the u or v terms or taken out as a factor.	3.1a	M1	$\int 4x\sin 2x\,dx$ $u = 4x$ $u' = 4$ $v' = \sin 2x$ $v = -\frac{1}{2}\cos 2x$ $\int 4x\sin 2x\,dx =$ $-\frac{4x}{2}\cos 2x - \int -\frac{4}{2}\cos 2x\,dx$ $= -2x\cos 2x + \sin 2x + c$ area above $\left[-2x\cos 2x + \sin 2x\right]_0^{\frac{\pi}{2}} = \pi$ area below $\left[-2x\cos 2x + \sin 2x\right]_{\frac{\pi}{2}}^{\pi} =$
	Substitutes their u, u', v, v' of **either** of the above forms into the integration by parts formula PI by $-2x\cos 2x + \sin 2x(+c)$	1.1a	M1	$-2\pi - \pi = -3\pi$
	Applies IBP correctly to obtain $\int 4x\sin 2x\,dx =$ $-\frac{4x}{2}\cos 2x - \int -\frac{4}{2}\cos 2x(dx)$ PI by $-2x\cos 2x + \sin 2x(+c)$	1.1b	A1	$\pi + 3\pi = 4\pi$
	Completes integration to obtain $-2x\cos 2x + \sin 2x(+c)$	1.1b	A1	
	Deduces $x = \frac{\pi}{2}$ at intersection with x-axis. Accept $\frac{\pi}{2}$ labelled on the diagram, written as a limit in their integral or seen as the solution to the equation $4x\sin 2x = 0$	2.2a	B1	
	Evaluates their integral over two separate intervals.	3.1a	M1	

Question 14 mark scheme continues on the next page

Q	Marking instructions	AO	Marks	Typical solution
14 cont	Demonstrates $\left[-2x\cos 2x+\sin 2x\right]_0^{\frac{\pi}{2}} = \pi$ **Or** $\left[-2x\cos 2x+\sin 2x\right]_{\frac{\pi}{2}}^{\pi} =$ $-2\pi-\pi=-3\pi$	1.1a	M1	
	Completes reasoned argument to show the area is 4π	2.1	R1	
	Question 14 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Obtains $\pm\dfrac{1}{2x}=\dfrac{y-2.5}{x-3}$ or $\pm 2x=\dfrac{y-2.5}{x-3}$ OE eg in the form of the equation of the straight line. **Or** Uses distance formula (may be in terms of x and y) $\left(d^2=\right)(x-3)^2+(y-2.5)^2$	3.1a	M1	$\dfrac{dy}{dx}=2x$ $-\dfrac{1}{2x}=\dfrac{y-2.5}{x-3}$ $-\dfrac{1}{2x}=\dfrac{x^2-2.5}{x-3}$ $-(x-3)=2x^3-5x$ $2x^3-4x-3=0$
	Obtains $-\dfrac{1}{2x}=\dfrac{y-2.5}{x-3}$ OE May have y replaced with x^2 **Or** Obtains $(x-3)^2+\left(x^2-2.5\right)^2$	3.1a	A1	
	Obtains $\pm\dfrac{1}{2x}=\dfrac{x^2-2.5}{x-3}$ OE Or $\pm 2x=\dfrac{x^2-2.5}{x-3}$ OE **Or** Differentiates their distance or distance[2] expression $\dfrac{d\left(d^2\right)}{dx}=2x-6+2\left(x^2-2.5\right)\times 2x$ $=4x^3-8x-6$	3.1a	M1	
	Completes reasoned argument to show $2x^3-4x-3=0$ Must see at least one line of correct intermediate working.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Obtains $6x^2 - 4$ May be explicit or seen as denominator in their $\dfrac{2x_n{}^3 - 4x_n - 3}{6x_n^2 - 4}$	1.1b	B1	$f(x) = 2x^3 - 4x - 3 = 0$ $f'(x) = 6x^2 - 4$ $x_{n+1} = x_n - \dfrac{2x_n{}^3 - 4x_n - 3}{6x_n^2 - 4}$ $= \dfrac{x_n\left(6x_n^2 - 4\right)}{6x_n^2 - 4} - \dfrac{2x_n{}^3 - 4x_n - 3}{6x_n^2 - 4}$ $= \dfrac{6x_n^3 - 4x_n - 2x_n{}^3 + 4x_n + 3}{6x_n^2 - 4}$ $= \dfrac{4x_n^3 + 3}{6x_n^2 - 4}$
	Forms $x_n - \dfrac{2x_n{}^3 - 4x_n - 3}{\text{Their "}6x_n^2 - 4\text{"}}$ Condone missing or inconsistent subscripts FT their $f'(x) = ax^2 - 4$ if stated explicitly	1.1a	M1	
	Obtains two correct fractions eg $\dfrac{x_n\left(6x_n^2 - 4\right)}{6x_n^2 - 4} - \dfrac{2x_n{}^3 - 4x_n - 3}{6x_n^2 - 4}$ **Or** obtains a single correct fraction with five terms in the numerator Condone missing or inconsistent subscripts	3.1a	A1	
	Completes reasoned argument with a **single correct fraction** before obtaining $x_{n+1} = \dfrac{4x_n^3 + 3}{6x_n^2 - 4}$ Condone missing or inconsistent subscripts but $x_{n+1} = \dfrac{4x_n^3 + 3}{6x_n^2 - 4}$ must be stated. AG	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Obtains 2.22 or AWRT 1.829 or AWRT 1.709 For 2.22 accept 2.220 or correct fraction $\frac{111}{50}$	1.1a	M1	$x_1 = 2.22$ $x_2 = 1.828840...$ $x_3 = 1.709$
	Obtains AWRT 1.709 as their final answer.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
15(d)	Uses expression for distance with their x_3 or a better approximation.	3.1a	M1	$\sqrt{(1.709-3)^2+(1.709^2-2.5)^2}$ $=1.3578...$
	Deduces AWRT 1.36	1.1b	A1	$=1.36$
	Subtotal		**2**	

	Question 15 Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Explains that the **sum** of the **angles** in a **triangle** is $\boldsymbol{\pi}$ (radians) **and** verifies *BAC* = $\frac{2\pi}{3} - x$ Eg $\pi = \frac{2\pi}{3} - x + \frac{\pi}{3} + x$ OE	2.4	E1	The angles in a triangle add up to π radians. Angle *BAC* = $\pi - \frac{\pi}{3} - x = \frac{2\pi}{3} - x$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Forms an equation using the sine rule using any two of $\dfrac{AC}{\sin\frac{\pi}{3}} = \dfrac{BC}{\sin A} = \dfrac{AB}{\sin x}$ OE Could use corresponding lowercase letters or letters defined or on the diagram throughout.	3.1a	M1	$\dfrac{BC}{\sin\left(\frac{2\pi}{3} - x\right)} = \dfrac{AB}{\sin x}$ $\sin\left(\frac{2\pi}{3} - x\right) = \sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x$ $= \frac{\sqrt{3}}{2}\cos x - -\frac{1}{2}\sin x$ $= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$ $\dfrac{BC}{AB} = \dfrac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\sin x}$ $= \dfrac{\sqrt{3}\cot x + 1}{2}$
	Obtains $\dfrac{BC}{\sin\left(\frac{2\pi}{3} - x\right)} = \dfrac{AB}{\sin x}$ Accept $\dfrac{BC}{\sin\left(\pi - \left(\frac{\pi}{3} + x\right)\right)} = \dfrac{AB}{\sin x}$ OE	1.1b	A1	
	Uses compound angle formula to expand $\sin\left(\frac{2\pi}{3} - x\right)$ or $\sin\left(\frac{\pi}{3} + x\right)$ Condone one sign error	3.1a	M1	
	Substitutes correct exact values for $\sin\left(\frac{2\pi}{3}\right)$ and $\cos\left(\frac{2\pi}{3}\right)$ Or for $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ into their expanded compound angle	1.1a	M1	
	Completes reasoned argument to obtain $\dfrac{BC}{AB} = \dfrac{\sqrt{3}\cot x + 1}{2}$ Accept $\dfrac{a}{c} = \dfrac{\sqrt{3}\cot x + 1}{2}$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	Deduces $\frac{\pi}{4}$ from $\frac{\sqrt{3}\cot x+1}{2}$	2.2a	R1	$\frac{\pi}{4}$
	Subtotal		**1**	

	Question 16 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)	Obtains $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ OE	1.1b	B1	$u=\mathrm{e}^x+1$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ $\int\frac{\mathrm{e}^{2x}}{\mathrm{e}^x+1}\mathrm{d}x=\int\frac{\mathrm{e}^{2x}}{u}\frac{1}{\mathrm{e}^x}\mathrm{d}u$ $=\int\frac{\mathrm{e}^x}{u}\mathrm{d}u$ $=\int\frac{u-1}{u}\mathrm{d}u$ $=\int 1-\frac{1}{u}\mathrm{d}u$ $=u-\ln u+c$ $=\mathrm{e}^x+1-\ln\left(\mathrm{e}^x+1\right)+c$ $=\mathrm{e}^x-\ln\left(\mathrm{e}^x+1\right)+k$
	Makes a partial substitution to obtain $\int\frac{\mathrm{e}^{2x}}{u}\frac{1}{\mathrm{e}^x}\mathrm{d}u$ or better If they simplify the fraction first $\int \mathrm{e}^x-\frac{\mathrm{e}^x}{\mathrm{e}^x+1}\mathrm{d}x$ need to see $\int \mathrm{e}^x-\frac{\mathrm{e}^x}{u}\frac{1}{\mathrm{e}^x}\mathrm{d}u$ or better for this mark	3.1a	M1	
	Obtains $\int\frac{u-1}{u}\mathrm{d}u$ OE	1.1b	A1	
	Integrates their two-term integrand, which is in terms of u, with at least one of their terms integrated correctly.	1.1a	M1	
	Completes reasoned argument to obtain $\mathrm{e}^x-\ln\left(\mathrm{e}^x+1\right)+k$ Must see constant of integration introduced as they integrate and dealt with consistently AG	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)	Obtains $\frac{1}{\cos^2 y}\frac{dy}{dx} = \frac{e^{2x}}{e^x + 1}$ or better	3.1a	B1	$\int \sec^2 y\, dy = \int \frac{e^{2x}}{e^x + 1} dx$ $\tan y = e^x - \ln\left(e^x + 1\right) + k$ $\tan \pi = e^0 - \ln\left(e^0 + 1\right) + k$ $k = \ln(2) - 1$ $\tan y = e^x + \ln\left(\frac{2}{e^x + 1}\right) - 1$
	Integrates to obtain one correct side. Uses given answer from **(a)** or recalls $\int \sec^2 y\, dy = \tan y$	1.1a	M1	
	Obtains $\tan y = e^x - \ln\left(e^x + 1\right) + k$ Condone missing or extra constants of integration.	1.1b	A1	
	Substitutes $y = \pi$ and $x = 0$ into their integrated equation which must be formed from exponential functions of x and trigonometric function(s) of y to obtain the constant of integration	1.1a	M1	
	Completes reasoned argument to obtain $\tan y = e^x + \ln\left(\frac{2}{e^x + 1}\right) - 1$	2.1	R1	
	Subtotal		**5**	

	Question 17 Total		**10**	

	Question Paper Total		**100**	