GCE

Mathematics A

H240/03: Pure Mathematics and Mechanics

A Level

Mark Scheme for June 2024

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2024

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$6a^5$	B2 [2]	1.1 1.1	**B1** for 6 **or** a^5 in their final answer **or** for $8a^6 \times \frac{3}{4}a^{-1}$	$\frac{24}{4}a^5$ as a final answer is **B1 B0**
1	(b)		$\frac{(2x-3)(2x+3)}{(2x-3)(x+4)(2x+3)}$ $\frac{1}{x+4}$	M1 A1 [2]	1.1 1.1	**M1** for either $4x^2-9=(2x-3)(2x+3)$ **or** $2x^2+5x-12=(2x-3)(x+4)$ oe, for example, $(x+4)^{-1}$	A correct answer www implies **M1** Do not ISW if further incorrect simplification occurs e.g. $\frac{1}{x+4}=\frac{1}{x}+\frac{1}{4}$ is **A0**

Question			Answer	Marks	AO	Guidance	
2			**DR**				**For reference only:**
			$\frac{1}{2}(3x+1)^2(2)$ or $(3x+1)^2$	**B1***	**1.1**	Correct use of $A=\frac{1}{2}r^2\theta$	$\frac{1}{2}(3x+1)^2(2)<44x-7$
			$9x^2+6x+1<44x-7 \Rightarrow 9x^2-38x+8\ (<0)$	**B1dep***	**1.1**	Expand and re-arrange to correct 3TQ expression in x	Allow **any** inequality sign or equals
			$9x^2-38x+8(<0) \Rightarrow (9x-2)(x-4)(<0)$	**M1dep***	**1.1**	**SEE APPENDIX** for awarding this mark (solving 3TQ expressions) - dependent on first **B** mark only (this mark is for solving **their** 3TQ but not for solving $(3x+1)^2=0$)	Correct quadratic followed immediately by correct critical values (with no working) is **M0**
			c.v. of x are $\frac{2}{9}, 4$	**B1**	**1.1**	Correct critical values of x (if factorisation shown then it must imply these two c.v.)	Must be $\frac{2}{9}$ or $0.\dot{2}$ but **B0** for 0.222…
			$\left\{x:\frac{2}{9}<x<4\right\}$	**B1FT dep***	**2.5**	FT their **two positive** critical values x_1, x_2 e.g. $\{x: x_1<x<x_2\}$ where $x_2>x_1$ allow $\left\{x:x>\frac{2}{9}\right\}\cap\{x:x<4\}$ but 'union' is **B0** **B0** for interval notation e.g. $\left(\frac{2}{9}, 4\right)$	Answer must be in set notation for this mark – **dependent on first B mark only**
				[5]			

Question			Answer	Marks	AO	Guidance	
3	(a)		$(3-2x)^{-2}=\frac{1}{9}(1+\ldots)^{-2}$	B1	1.1a	For reference: $\frac{1}{9}\left(1-\frac{2}{3}x\right)^{-2}$ - soi	or for $3^{-2}(1+...)^{-2}$
			$(1+kx)^{-2}=1+(-2)(kx)+...$	B1FT	1.1	Correct first two terms follow through their k – allow un-simplified	$k\neq\pm1,\pm2$ - if correct $k=-\frac{2}{3}$
			$...+\frac{(-2)(-3)}{2!}(kx)^2$	B1FT	1.1	Correct third term following through their k – allow un-simplified but must imply that the third term contains their k^2 - for correct k condone $\frac{2\times3}{2!}\left(\frac{2}{3}x\right)^2$ (or similar for their k if negative)	$k\neq\pm1,\pm2$ Condone $\frac{2\times3}{2!}(kx)^2$ and 2 for 2!
			$(3-2x)^{-2}=\frac{1}{9}\left(1+\frac{4}{3}x+\frac{4}{3}x^2+...\right)$	B1	1.1	Or correct equivalent e.g. $\frac{1}{27}\left(3+4x+4x^2\right)$, $\frac{1}{9}+\frac{4}{27}x+\frac{4}{27}x^2$, etc. ISW after correct expansion seen	Ignore higher order terms if found – a correct answer scores all 4 marks www
				[4]			

Question			Answer	Marks	AO	Guidance	
3	(b)		$\lvert x\rvert < \frac{3}{2}$	B1 [1]	2.5	oe, for example, $-\frac{3}{2} < x < \frac{3}{2}$ - allow $-\frac{3}{2} \leqslant x < \frac{3}{2}$ but not $-\frac{3}{2} \leqslant x \leqslant \frac{3}{2}$ (or any inequality that includes the $\frac{3}{2}$) - ISW once correct inequality seen. **Allow** $\left[-\frac{3}{2}, \frac{3}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ oe but not $\left[0, \frac{3}{2}\right)$ (or equivalents in set notation)	$-\frac{3}{2} < \lvert x\rvert < \frac{3}{2}$ is **B0** but $0 \leqslant \lvert x\rvert < \frac{3}{2}$ is **B1** **Note that** $\lvert 2x\rvert < 3$ **only is B0** (must be in terms of x)
3	(c)		$\frac{a+x}{(3-2x)^2} = (a+x)\left(\frac{1}{9} + \frac{4}{27}x + ...\right)$ $= ... + \left(\frac{1}{9} + \frac{4}{27}a\right)x + ...$ $\frac{4}{3}a + 1 = 0 \Rightarrow a = -\frac{3}{4}$	B1FT B1FT [2]	3.1a 2.2a	Finding correct coefficient of x **or** the x term for their $(p + qx + ...)(a + x)$ - **FT** their p and q from part **(a)** (so their x-coefficient must be $p + aq$). Allow embedded in an expansion e.g. $= \frac{1}{9}\left(... + \left(\frac{4}{3}a + 1\right)x + ...\right)$ or $= \frac{1}{9}\left(... + \frac{4}{3}ax + x + ...\right)$ Follow through $-\frac{\textit{their}\text{ constant term}}{\textit{their}\text{ coefficient of } x}$ from part **(a)**	This mark can be implied by the correct answer for a (or on the **FT** as detailed in the next mark)

Question			Answer	Marks	AO	Guidance	
4	(a)		$2\cot^2 x - 9\operatorname{cosec} x - 3[=0]$ $2\left(\frac{\cos^2 x}{\sin^2 x}\right) - 9\left(\frac{1}{\sin x}\right) - 3[=0]$	**M1**	**2.1**	Use of both $\cot x \equiv \frac{\cos x}{\sin x}$ and $\operatorname{cosec} x \equiv \frac{1}{\sin x}$	Condone use of *s* and *c* throughout but final answer must be in terms of sine – allow e.g. θ for x for **M** marks but must be x for the **A** mark
			$2\cos^2 x - 9\sin x - 3\sin^2 x[=0]$ $2\left(1-\sin^2 x\right) - 9\sin x - 3\sin^2 x[=0]$	**M1**	**1.1**	Correct use of $\sin^2 x + \cos^2 x \equiv 1$ to obtain an equation in $\sin x$ only	**Not dependent on the first M mark**
			$2 - 2\sin^2 x - 9\sin x - 3\sin^2 x[=0]$ $\Rightarrow 5\sin^2 x + 9\sin x - 2 = 0$	**A1**	**2.2a**	**AG** (so must be equal to zero) – sufficient working must be shown – any errors seen is **A0**	**A0** if an angle is missing from any trig. expression **used in their working**
				[3]			
			Alternative method				
			$2(\operatorname{cosec}^2 x - 1) - 9\operatorname{cosec} x - 3[=0]$	**M1**		Correct use of $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$	
			$2\operatorname{cosec}^2 x - 9\operatorname{cosec} x - 5[=0]$ $\Leftrightarrow \frac{2}{\sin^2 x} - \frac{9}{\sin x} - 5[=0]$	**M1**		Replacing $\operatorname{cosec} x$ with $\frac{1}{\sin x}$ to obtain an equation in $\sin x$ only Note: $2\operatorname{cosec}^2 x - 9\operatorname{cosec} x - 5 = 0$ $\Rightarrow 2 - 9\sin x - 5\sin^2 x = 0$ with no intermediate working is **M0** unless explicit mention is made of multiplying through by $\sin^2 x$	**Not dependent on first M mark** or explicit mention is made of dividing by $\operatorname{cosec}^2 x$
			$2 - 9\sin x - 5\sin^2 x[=0]$ $\Rightarrow 5\sin^2 x + 9\sin x - 2 = 0$	**A1**		**AG** (so must be equal to zero) – sufficient working must be shown – any errors seen is **A0**	**A0** if an angle is missing from any trig. expression used in their working

Question			Answer	Marks	AO	Guidance	
4	(b)	(i)	**DR**				
			$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3[= 0]$ $\Rightarrow (5\sin 2\theta - 1)(\sin 2\theta + 2)[= 0]$	**M1**	**1.1**	**SEE APPENDIX for awarding this mark (solving 3TQ expressions)**	condone using x for θ **or** 2θ for the **M** mark – condone for **M1** only $(5\sin\theta - 1)(\sin\theta + 2)$
			$\sin 2\theta = 0.2$ only as $\sin 2\theta \neq -2$	**B1**	**2.3**	Correctly stating that $\sin 2\theta = 0.2$ and that $\sin 2\theta$ cannot equal -2 (must explicitly reject the -2 (but no rationale required) - this mark is not implied by correct values for θ (as **DR** required)	Must be solving $5\sin^2 2\theta + 9\sin 2\theta - 2 = 0$ for the **B** marks condone $\sin 2x = 0.2$
			$[\theta =] 0.101$	**B1**	**1.1**	awrt 0.101 (0.1006789…) www	
			$[\theta =] 1.470$	**B1**	**1.1**	awrt 1.470 (1.4701173…) www	
						Ignore additional solutions outside of the range $0 < \theta < \pi$, but if any other solutions inside the range, award at most one of the two final **B** marks for one correct value	**SC B1** for awrt 0.10 **and** awrt 1.47 only if 3 dp or better) not seen **SC B1** for awrt 5.77 **and** awrt 84.2 only (working in degrees)
				[4]			

Question			Answer	Marks	AO	Guidance	
4	(b)	(ii)	$5\sin^2 2\theta + 9\sin 2\theta - 2[=0]$ $\Rightarrow 5(2\theta)^2 + 9(2\theta) - 2[=0]$ $\left(10\theta^2 + 9\theta - 1[=0]\right)$	M1	1.2	Use of the small angle approximation $\sin 2\theta \approx 2\theta$ twice in the given answer from **(a)** to obtain a three-term quadratic in θ (allow un-simplified)	Award **M1** only for $5\theta^2 + 9\theta - 2[=0]$ (so for using θ instead of 2θ) – allow e.g. x for θ
			$(10\theta - 1)(\theta + 1) = 0 \Rightarrow \theta = 0.10(000...)$ so is accurate to 2 decimal places	A1 [2]	2.4	State 0.10 (or better e.g. 0.100…) **as a decimal** following a correct quadratic in θ seen (no method required for solving the quadratic) and comment that this is accurate to 2 dp (**as a minimum** must mention '2 dp' with the value of 0.10(000…) appearing in this part and 0.10 **or** 0.101 **or** 0.100(6789…) appearing in part **(b)(i)**)	**This mark is dependent on an awrt 0.10 seen in part (b)(i) or a correct sign change test (see below)** **Ignore any consideration of other root(s)** **SEE APPENDIX FOR ALTERNATIVE**
			Alternative for M mark $\sin 2\theta = 0.2$ (from part **(b)(i)**) $\Rightarrow 2\theta = 0.2$			Re-writing at least one of their equations $\sin 2\theta = k$ with $-1 < k < 1$ (from part **(a)**) as $2\theta = k$	
			Alternative for A mark $f(\theta) = 2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3$ $f(0.105) = -2.1497... < 0$ $f(0.095) = 3.4185... > 0$ Change of sign indicates that the approximate solution is accurate to 2 decimal places			Correct values to at least 1 dp (rot) with explanation ('change of sign' either stated **or** comparing values with zero) **and** correct conclusion (as a minimum must mention '2 dp')	

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	$y=\left(x^3-2x^2\right)\ln x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=...$	**M1**	**2.1**	**M1** for attempt to differentiate using the product rule (oe) – answer must be of the form $\left(x^3-2x^2\right)\times\frac{k_1}{x}+\left(k_2x^2+k_3x\right)\ln x$ for non-zero constants k_1, k_2, k_3	Condone invisible brackets for this mark
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^3-2x^2\right)\left(\frac{1}{x}\right)+\left(3x^2-4x\right)\ln x$	**A1**	**1.1**	**A1** for a correct first derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2x-2+\frac{3x^2-4x}{x}+(6x-4)\ln x$	**A1**	**1.1**	**A1** for a correct second derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0 \Rightarrow 2x-2+\frac{3x^2-4x}{x}+(6x-4)\ln x=0$	**M1**	**1.1**	Setting the second derivative (which if simplified would be of the form $ax+b+(cx+d)\ln x$ with non-zero constants a, b, c and d) equal to zero	
			$5x-6=(4-6x)\ln x \Rightarrow x=\frac{6+(4-6x)\ln x}{5}$	**A1**	**2.2a**	**AG** – so sufficient working must be shown – at least one intermediate line of working from second derivative set equal to zero to given answer	**Any** errors seen (e.g. any missing/invisible brackets) is **A0**
				[5]			

Question			Answer	Marks	AO	Guidance	
5	(a)	(ii)	$x_{n+1} = \frac{6+(4-6x_n)\ln x_n}{5}$ $x_1 = 1.1$ $x_2 = 1.150438...$ $x_3 = 1.118643...$ $x_4 = 1.139191...$ $x_5 = 1.126105...$ $x_6 = 1.134521...$	**B1**	**1.1**	Uses given result and given starting value (of 1.1) to obtain correct x_2 and x_3 (so first two iterations after the initial value of 1.1) to at least 2 dp (rot) – but all stated values in these two terms must be correct	
			x-coordinate of M is 1.13	**B1** **[2]**	**2.2a**	Must be stated to **2** dp only – **not dependent on the first B mark** – can be awarded if either of x_2 and x_3 are incorrect (assume that the iterative process corrected itself or a slip in the candidate writing down an earlier value)	Must be clear that x is 1.13 (and not the final term shown in the iterative process e.g. $x_6 = 1.13$ only is **B0**) – this mark can be awarded from using alternative iterative methods e.g. Newton-Raphson

Question			Answer	Marks	AO	Guidance	
5	(b)		Curve crosses the x-axis at 1 and 2	B1*	3.1a	Correct x-intercepts (soi) – ignore mention of $x = 0$	Could be seen as limits on integral(s)
			$\int\left(x^3-2x^2\right)\ln x\,\mathrm{d}x = ...$	M1*	2.1	**M1** for attempt at integration by parts – **must be of the form** $(ax^4+bx^3)\ln x \pm \int (cx^4+dx^3)\times\frac{1}{x}(\mathrm{d}x)$ for non-zero constants a, b, c and d	Limits not required for this and the next two **A** marks (so condone incorrect limits too for these 3 marks)
			$=\left(\frac{1}{4}x^4-\frac{2}{3}x^3\right)\ln x-\int\left(\frac{1}{4}x^4-\frac{2}{3}x^3\right)\left(\frac{1}{x}\right)\mathrm{d}x$	A1	1.1	correct first application (allow un-simplified)	dx not required and integral sign(s) can be implied
			$=\left(\frac{x^4}{4}-\frac{2x^3}{3}\right)\ln x-\frac{x^4}{16}+\frac{2x^3}{9}(+c)$	A1	1.1	cao (allow un-simplified)	
			$\left\{\left(\frac{16}{4}-\frac{16}{3}\right)\ln 2-1+\frac{16}{9}\right\}-\left\{0-\frac{1}{16}+\frac{2}{9}\right\}$	M1dep*	1.1	Uses correct limits completely correctly $\pm(F(2)-F(1))$ in **their** fully integrated expression – need not be simplified (or equivalent)	Do not condone invisible brackets unless recovered
			$\int_1^2\left(x^3-2x^2\right)\ln x\,\mathrm{d}x = -\frac{4}{3}\ln 2+\frac{89}{144}$ $\Rightarrow \text{Area} = \frac{4}{3}\ln 2-\frac{89}{144}$	A1	3.2a	Must be of this form but allow exact equivalents (for example, $\frac{1}{3}\ln 16-\frac{89}{144}$) but the p and r must be positive rational numbers and q must be a positive integer **Be aware of those who consider** $\int_1^2(2x^2-x^3)\ln x\,\mathrm{d}x$ **which is correct**	For reference – one possibility is: $p=\frac{4}{3}, q=2, r=\frac{89}{144}$ **These values do not need to be stated explicitly**
				[6]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$x = 4k + k\sin t, y = 2 + 4\cos t$ $\sin t = \frac{x-4k}{k}, \cos t = \frac{y-2}{4}$ and use of $\sin^2 t + \cos^2 t = 1$	M1	3.1a	Re-arranges to obtain both $\sin t = \frac{x \pm 4k}{k}$ **and** $\cos t = \frac{y \pm 2}{4}$ **and** use $\sin^2 t + \cos^2 t = 1$ to eliminate t	
			$\frac{(x-4k)^2}{k^2} + \frac{(y-2)^2}{16} = 1$	A1 [2]	1.1	Allow any correct un-simplified cartesian form not involving trigonometric terms	ISW once a correct answer seen

SC for part **(a)**: If **M0** awarded then **SC B1** for $y = 2 + 4\cos\left(\arcsin\left(\frac{x-4k}{k}\right)\right)$ **or** $x = 4k + k\sin\left(\arccos\left(\frac{y-2}{4}\right)\right)$ **or** $\arcsin\left(\frac{x-4k}{k}\right) = \arccos\left(\frac{y-2}{4}\right)$ (or any correct form in terms of trig. functions – allow $\cos^{-1}$ for arccos etc.)

If considering one branch only e.g. $y(= 2 + 4\sqrt{1-\sin^2 t}) = 2 + 4\sqrt{1-\left(\frac{x-4k}{k}\right)^2}$ this scores **M1** only (or equivalent expression for x)

If considering both branches e.g. $y(= 2 \pm 4\sqrt{1-\sin^2 t}) = 2 \pm 4\sqrt{1-\left(\frac{x-4k}{k}\right)^2}$ then this scores **M1 A1** (or equivalent expression for x)

Note a (common) correct answer for **M1 A1** is: $x^2 + y^2 = 16k^2 + 8k(x-4k) + (x-4k)^2 + 4 + 4(y-2) + (y-2)^2$

Question			Answer	Marks	AO	Guidance	
6	(b)	(i)	C is a circle $\Rightarrow k^2 = 16$	M1	3.1a	Setting c and d equal in their $\frac{(x \pm a)^2}{c} + \frac{(y \pm b)^2}{d} = 1$ **or** stating that $y_{\min} = -2$ **and** $y_{\max} = 6$	Possibly implied by correct value for r www
			$r = 4$	A1	1.1	$r = \pm 4$ is **A0** unless replaced with positive r only	
				[2]			
6	(b)	(ii)	$(16, 2), (-16, 2)$	B2	2.2a 2.2a	**B1** for either one correct centre **or** both x-coordinates correct	**These marks are dependent on $r = 4$ from correct working or www**
				[2]			

Question			Answer	Marks	AO	Guidance	
7			For reference: $5x-2xy+2y^2-k=0$	**M1***	**2.1**	Either $\frac{\mathrm{d}}{\mathrm{d}x}(2xy)=2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right)=2y\frac{\mathrm{d}y}{\mathrm{d}x}$	Attempt at implicit differentiation wrt x
			$5-2y-2x\frac{\mathrm{d}y}{\mathrm{d}x}+4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**A1**	**1.1**	Correct equation (any form) – allow = 0 implied by later working	Setting derivative equal to y' is **A0**
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2y-5}{4y-2x}$ so at P and Q, $4y-2x=0$	**M1dep***	**3.1a**	Set the denominator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (oe)	Denominator must be linear in both x and y
			$5x-2xy+2y^2-k\ [=0]$ $\Rightarrow 5(2y)-2(2y)y+2y^2-k\ [=0]$ $\Rightarrow 2y^2-10y+k\ [=0]$	**M1**	**1.1**	Eliminate x correctly and simplify to a 3TQ quadratic in y (if correct: $2y^2-10y+k=0$) **or** eliminate y correctly to form a 3TQ quadratic in x (if correct: $x^2-10x+2k=0$)	Dependent on both previous **M** marks – condone sign slips only in simplification
			Difference in y values is 3 so $\frac{\sqrt{100-8k}}{2}=3$ $\left(\text{or } \frac{10+\sqrt{100-8k}}{4}-\frac{10-\sqrt{100-8k}}{4}=3\right)$	**M1**	**1.1**	Setting up the equation for diff. in y $\frac{\sqrt{b^2-4ac}}{a}=\pm 3$, LHS of equation must be correct following through from their 3TQ (the constant term for their 3TQ in y must contain k)	Dependent on first two **M** marks – if considering diff. in x values = 3 then **M0** **Correct *k* or both *y's* www is M1 A1**
			$k=8$	**A1**	**1.1**	www (can be implied by correct y-values)	$y_P=1, y_Q=4$
			$x_P=2,\ x_Q=8$	**A1**	**2.2a**	Both correct www – do not penalise $x_P=8,\ x_Q=2$	**Dependent on all previous marks**
				[7]			

Question	Answer	Marks	AO	Guidance	
	Alternative for the first 3 marks			**Implicit differentiation wrt *y***	
	Attempt at implicit differentiation wrt y	**M1**		Either $\frac{\mathrm{d}}{\mathrm{d}x}(2xy) = 2x + 2y\frac{\mathrm{d}x}{\mathrm{d}y}$ **or** $\frac{\mathrm{d}}{\mathrm{d}y}(5x) = 5\frac{\mathrm{d}x}{\mathrm{d}y}$	
	$5\frac{\mathrm{d}x}{\mathrm{d}y} - 2x - 2y\frac{\mathrm{d}x}{\mathrm{d}y} + 4y = 0$	**A1**		Correct equation (any form) – allow = 0 implied by later working	Setting derivative equal to x' is **A0**
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{4y-2x}{2y-5}$ so at P and Q, $4y - 2x = 0$	**M1**		Set the numerator of their $\frac{\mathrm{d}x}{\mathrm{d}y} = 0$ oe	
	Alternative for the first 4 marks			**Making *x* the subject, diff. wrt *y***	
	$5x - 2xy + 2y^2 - k = 0 \Rightarrow x = \frac{k-2y^2}{5-2y}$	**M1***		Makes x the subject – must be of the form $x = \frac{\mathrm{f}(y)}{\mathrm{g}(y)}$	where $\mathrm{f}(y)$ is a two-term quadratic fn. in y and $\mathrm{g}(y)$ is a two-term linear fn. in y
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{(5-2y)(-4y)-(k-2y^2)(-2)}{(5-2y)^2}$	**A1**		Correct derivative (allow un-simplified)	Any equivalent correct form
	$(5-2y)(-4y)-(k-2y^2)(-2) = 0$	**M1dep***		Set the numerator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero (oe) where $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\mathrm{g}(y)\times\mathrm{f}'(y) \pm \mathrm{f}(y)\times\mathrm{g}'(y)}{(\mathrm{g}(y))^2}$	Where f(y) and g(y) are as defined in the first **M** mark and derivatives follow through correctly from their f and g
	$-20y + 8y^2 + 2k - 4y^2 = 0$ $\Rightarrow 2y^2 - 10y + k = 0$	**M1**		Simplify to a 3TQ quadratic in y (if correct: $2y^2 - 10y + k = 0$)	Dependent on both previous **M** marks – condone sign slips only in simplification to 3 TQ in y

Question			Answer	Marks	AO	Guidance	
			Alternative 1 for the fourth M mark			**Roots of polynomials**	
			If correct: $2y^2-10y+k\,[=0]$ Difference in y values is 3 therefore if the y roots are $\alpha, 3+\alpha$ then $\alpha+(3+\alpha)=-\dfrac{-10}{2}$ $(\alpha=1\Rightarrow y=1,4)$ **If using this method, then candidates do not need to find *k*, so the penultimate A mark is for the correct two *y* values (1 and 4)**	**M1**		Using $\sum\alpha=-\dfrac{b}{a}$ correctly for their 3TQ in y with roots that differ by 3	Could use e.g. $\alpha, \alpha-3$ etc. Dependent on first two **M** marks
			Alternative 2 for the fourth M mark			**Difference in *x*-values**	
			Difference in y-values is 3 therefore the difference in x values is 6 (from $4y-2x=0$) Therefore, as $x^2-10x+2k=0$ $\Rightarrow\sqrt{100-8k}=6$ $\left(\text{or } \dfrac{10+\sqrt{100-8k}}{2}-\dfrac{10-\sqrt{100-8k}}{2}=6\right)$	**M1**		Setting up the equation for diff. in x $\dfrac{\sqrt{b^2-4ac}}{a}=\pm\text{their 6, with their 6}$ from their $4(3)-2x=0$, LHS of equation must be correct following through from their 3TQ (the constant term for their 3TQ in x must contain k)	Dependent on first two **M** marks

Question			Answer	Marks	AO	Guidance	
8	(a)		$[\mathbf{s}=]\ 2(\mathbf{i}+3\mathbf{j})+0.5\times 2^2\times(-5\mathbf{i}+2\mathbf{j})$ or $2\begin{pmatrix}1\\3\end{pmatrix}+0.5\times 2^2\times\begin{pmatrix}-5\\2\end{pmatrix}$	**M1**	**3.3**	Apply $\mathbf{s}=\mathbf{u}t+0.5\mathbf{a}t^2$ correctly with correct values of **u**, **a** and t – if using integration then for this mark we must see the correct expression $\begin{pmatrix}1\\3\end{pmatrix}t+\frac{1}{2}\times\begin{pmatrix}-5\\2\end{pmatrix}t^2$ **with** $t=2$ subst.	
			$[\mathbf{s}=]-8\mathbf{i}+10\mathbf{j}$ (m)	**A1**	**1.1**	or $\begin{pmatrix}-8\\10\end{pmatrix}$	ISW if correct vector converted to scalar
				[2]			
8	(b)		$\mathbf{v}=(\mathbf{i}+3\mathbf{j})+2(-5\mathbf{i}+2\mathbf{j})$	**M1***	**3.3**	Apply $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ with correct values of **u**, **a** and t (or other complete method to find **v**)	Allow from integration but must have correct expression for **v** with $t=2$ substituted
			$\mathbf{v}=-9\mathbf{i}+7\mathbf{j}$	**A1**	**1.1**	or as a column vector (possibly implied by correct magnitude)	
			$\lvert\mathbf{v}\rvert=\sqrt{(-9)^2+7^2}$	**M1dep***	**3.4**	Correct method for the speed of P at time $t=2$ – condone $\sqrt{-9^2+7^2}=\sqrt{\pm 81+49}$	
			$\lvert\mathbf{v}\rvert=11.4\ (\text{m s}^{-1})$	**A1**	**1.1**	Allow $\sqrt{130}$ or awrt 11.4 www – must follow from correct $\mathbf{v}=-9\mathbf{i}+7\mathbf{j}$ (so **M1 A0 M1 A1** is not possible)	11.4017542…
				[4]			

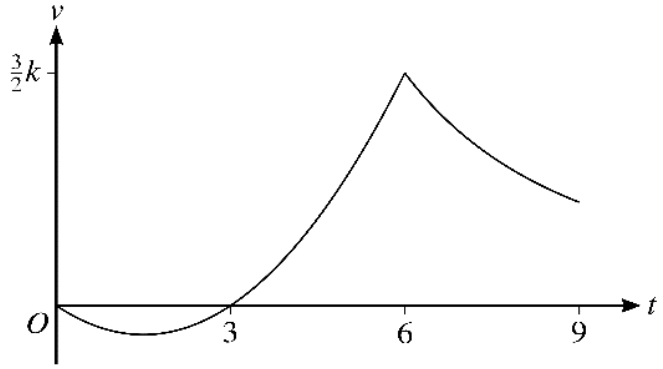
Question			Answer	Marks	AO	Guidance	
9	(a)			**M1***	**2.1**	Resolve vertically **or** horizontally – correct number of relevant terms. Allow sign errors and sin/cos mix but must be using correct angles	
			$R(\uparrow): \pm(12\cos 20 + 17\cos 50)$	**A1**	**1.1**	Need not be simplified	$\pm 22.20370081\ldots$
			$R(\rightarrow): \pm(17\sin 50 - 12\sin 20)$	**A1**	**1.1**	Need not be simplified	$\pm 8.918513813\ldots$
			Magnitude is $\sqrt{(12\cos 20 + 17\cos 50)^2 + (17\sin 50 - 12\sin 20)^2}$	**M1dep***	**3.4**	Correct method to calculate magnitude from expressions with the correct number of relevant terms	
			$\tan\theta = \dfrac{17\sin 50 - 12\sin 20}{12\cos 20 + 17\cos 50}$	**M1dep***	**3.4**	Correct method to calculate a relevant angle (so allow reciprocal) from expressions with the correct number of relevant terms	
			Resultant force is 23.9 (N) on a bearing of 022(°)	**A1**	**2.2a**	awrt 23.9 **and** awrt 022 (so 021.9 is okay but 21.9 is **A0**)	23.927896… 21.883744…
			Alternative method				
			$R^2 = 17^2 + 12^2 - 2(17)(12)\cos\alpha$	**M1***		Correct cosine rule with any angle α	
			$R^2 = 17^2 + 12^2 - 2(17)(12)\cos 110$	**B1 A1**		**B1** for correct angle of 110 seen, **A1** for correct expression for R or R^2	110 may be seen on a diagram
			$\dfrac{\sin\theta}{12} = \dfrac{\sin\alpha}{R}$ or $\cos\theta = \dfrac{R^2 + 17^2 - 12^2}{2\times R\times 17}$	**M1dep***		Correct sine rule with their α or correct cosine rule	If correct $\theta = 28.11625...$
			Bearing = $50 - \theta$	**M1dep***		Correct calculation for bearing	**Dep. both M marks**
			Resultant force is 23.9 (N) on a bearing of 022(°)	**A1**		awrt 23.9 and awrt 022 (so 021.9 is okay but 21.9 is **A0**)	
				[6]			

Question			Answer	Marks	AO	Guidance	
9	(b)		Magnitude of **F** is 23.9 (N)	**B1FT**	**1.2**	Follow through their resultant in **(a)**	
			On a bearing of 202(°)	**B1FT**	**3.1b**	Follow through the answer to **(a)** – if $0 < \theta < 180$ then FT is $180 + \theta$ and if $180 < \theta < 360$ then FT is $\theta - 180$	**As in part (a) their bearing must be at least three figures (with leading zeros if necessary) but allow greater accuracy**
				[2]			

Question			Answer	Marks	AO	Guidance	
10	(a)		$R(\uparrow \text{object}):\quad T = 5g$	B1	1.1	Possibly implied by later working	Allow $T - 5g = 0$
			$R(\rightarrow \text{block}):\quad T\cos 50 = X\cos 20$	M1	3.3	Resolving horizontally to form an equation – correct number of terms (possibly with T replaced with $5g$) – allow sin/cos mix but must be using correct angles	Must be components of both T and X
			$X = 33.5$	A1	1.1	awrt 33.5 – condone including of 'N'	33.51797…
				[3]			
10	(b)		$X\sin 20 + 147 + T\sin 50 = mg$	M1	3.3	Resolving vertically to form an equation for the block with the correct number of relevant terms – allow sign errors and sin/cos mix but must be using correct angles	Allow X, T or their value(s) for X and T – **relevant means both dimensionally correct (so must be using *mg*) and that we must see components of *T* and *X* but no component of the weight or the 147**
			$('33.5...')\sin 20 + 147 + 5g\sin 50 = mg$	A1FT	1.1	Correct equation for m **FT** their value of X only (so not just X)	
			$m = 20$	A1	1.1	Accept either 20 or awrt 20.0; condone inclusion of 'kg'	www
				[3]			

Question			Answer	Marks	AO	Guidance	
11	(a)		$R_D \times k_1 \sin 50 = 0.5g \times k_2$ **or** $R_D \times k_1 \cos 50 = 0.5g \times k_2$ **(see second guidance column)**	**M1**	**3.3**	Attempt at moments about A – at least two terms (one term for the weight and one term with a component of R_D) – **must be written as an equation to score any marks in this part (see Answer column for the only acceptable forms)**	**with** $k_1, k_2 > 0$ **but not =** $\pm$ **1**
			$6(R_D \sin 50) = ...$	**A1**	**1.1**	Correct moment for the contact force at D	
			$...3(0.5g \cos 50) - 1(0.5g \sin 50)$	**A1**	**3.1b**	Correct moment for the weight of the lamina (**See Appendix**) (e.g. $0.5g \times (3 - \tan 50) \cos 50$ or $0.5g \times \cos 50 \times 1.808...$ etc. e.g. $0.5g \times \sqrt{10} \sin(40 - \arctan(\frac{1}{3}))$ or $0.5g \times \sqrt{10} \sin(21.565...)$ etc.) – if value stated with no or unclear working, then must see at least 5.6953… to award this mark - if moment stated as 5.6993… then **A0** (and **A0** for the next mark)	For reference 5.695360…
			$R_D = \dfrac{1.5g \cos 50 - 0.5g \sin 50}{6 \sin 50} = 1.24\,(\text{N})$	**A1**	**2.2a**	**AG – as this answer is given then working must be checked carefully – all previous marks must have been awarded**	1.23912743… **A fully correct equation followed by stating 1.24 can score full marks**
				[4]			

Question			Answer	Marks	AO	Guidance	
11	(b)		$R(\uparrow): R_A = 0.5g$ $R(\rightarrow): 1.24 = F_A$	**B1***	**3.3**	Resolving horizontally **and** vertically (possibly implied by later working) – must be using given 1.24 (or a more accurate correct value) only (so not their incorrect value from part **(a)**)	If taking moments about another point e.g. D, then the corresponding equation(s) must be correct
			$1.24 = 0.5g\mu$	**M1dep***	**3.4**	**Use of** $F = \mu R$ or $F \leqslant \mu R$ with $0.5g$ and 1.24 (or better) for F	
			$\mu = 0.253$	**A1**	**2.2a**	awrt 0.253 If final answer is $\mu \geqslant 0.253$ then **A0**	Using the exact value or 1.24 for R_D leads to the same answer to **3** significant figures
				[3]			

Question			Answer	Marks	AO	Guidance	
12	(a)			**B1** **B1** **B1** **[3]**	**1.1** **1.1** **1.1**	Quadratic curve (positive quadratic) passing through the origin and appearing in both quadrants with a minimum turning point in the fourth quadrant Inverse proportional (to t^2) curve (so correct curvature), should not touch the horizontal axis Completely correct graph with both curves meeting at 6 **with values of 3 and 6 on *t* axis** (or clearly stated in their working in part **(a) only**) – this mark is dependent on the previous 2 **B** marks	No scaling on axes required for the first two **B** marks (**so ignore any scaling for the first two marks**) Condone if it appears to be approaching a horizontal asymptote (other than the *t*-axis) for this mark but gradient must not be positive Allow only the *t*-axis as a horizontal asymptote **Value of 9 not required on *t* axis, neither is $\frac{3}{2}k$ on the *v*-axis**
12	(b)		1.5 (s) only	**B1** **[1]**	**1.1**	From symmetry or solving $\frac{1}{12}k(2t-3)=0$ www	Any other answer(s) is **B0**

Question			Answer	Marks	AO	Guidance	
12	(c)		**DR**				
			$[k]\frac{1}{12}\int\left(t^2-3t\right)\mathrm{d}t=\frac{1}{12}\left(\frac{t^3}{3}-\frac{3}{2}t^2\right)[k]$	**M1***	**2.1**	Attempt to integrate – **both terms with power increased by 1 with one term correct**	Ignore +*c* omission for **M** marks
			For **both** $[k]\frac{1}{12}\int_0^3\left(t^2-3t\right)\mathrm{d}t=-\frac{3}{8}[k]$ **or** $+\frac{3}{8}[k]$ **and** $[k]\frac{1}{12}\int_3^6\left(t^2-3t\right)\mathrm{d}t=\frac{15}{8}[k]$ (**but allow correct un-simplified expressions** – **SEE APPENDIX**)	**B1**	**1.1**	www Condone $[k]\frac{1}{12}\int_0^3\left(t^2-3t\right)\mathrm{d}t=+\frac{3}{8}[k]$ Award **B1** for $[k]\frac{1}{12}\int_0^6\left(t^2-3t\right)\mathrm{d}t=\frac{18}{8}[k]$ but **B0** for $[k]\frac{1}{12}\int_0^6\left(t^2-3t\right)\mathrm{d}t=\frac{12}{8}[k]$	The correct values do not imply the **M** mark as **DR**
			$[k]\int\frac{54}{t^2}\mathrm{d}t=-\frac{54}{t}[k]$	**M1***	**1.1**	Attempt to integrate – answer of the form ct^{-1} with $c\neq 1,54k,54$	
			$[k]\int_6^9\frac{54}{t^2}\mathrm{d}t=3[k]$ (**allow correct un-simplified** – **SEE APPENDIX**)	**B1**	**1.1**	www	The correct value does not imply the **M** mark as **DR**
			$\frac{3}{8}k+\frac{15}{8}k+3k=84$ or $\frac{18}{8}k+3k=84$	**M1dep***	**3.4**	Forming a linear equation in *k* with the correct number of relevant terms (e.g. must have taken the modulus of their integral from 0 to 3)	$\frac{1}{12}k\int_0^6\left(t^2-3t\right)\mathrm{d}t=\frac{12}{8}k$, or any working that suggests the modulus was not taken between 0 and 3 is **M0**
			$k=16$	**B1** **[6]**	**2.2a**	www	

Question			Answer	Marks	AO	Guidance	
13	(a)		$R(\text{perp. plane}): R = mg\cos 36.86...$ **or** $mg \times 0.8$	**B1**	**3.3**	Allow using an angle of 37 or better	soi (possibly in N2L)
			$F = 0.25 \times mg\cos 36.86...$ **or** $0.25 \times mg \times 0.8$	**M1***	**3.4**	$F = 0.25R$ with their R (with $R \neq mg$) – using an angle of 37 or better – allow sin/cos mix	R must be a component of weight soi (possibly in N2L)
			N2L (parallel to plane): $-0.25 \times mg \times 0.8 - mg \times 0.6 = ma$ or $-0.25 \times mg \times \cos 36.86... - mg \times \sin 36.86... = ma$	**M1dep***	**3.1b**	Applying N2L parallel to the plane with correct number of terms and weight term resolved, dimensionally consistent – if down the plane is the positive direction expect $0.25 \times mg \times 0.8 + mg \times 0.6 = ma$	Allow sign errors and sin/cos mix, using an angle of 37 or better
			$a = -7.84$ or 7.84	**A1**	**1.1**	$\pm 0.8g$ - allow awrt ± 7.84 or awrt ± 7.85 (using an angle of 37) www	Condone ± 7.8 or ± 7.9 www
			$v^2 = 6^2 + 2 \times -7.84 \times 1.375$	**M1dep***	**3.4**	Use of $v^2 = u^2 + 2as$ with correct u and s and their negative a $(\neq -9.8)$ - **dependent on both previous M marks – must see the values substituted into the formula (as value for v is AG) – therefore just seeing $v^2 = \frac{361}{25}$ or 14.44 is M0**	Condone finding a positive value of a and then using the negative version
			$v^2 = 14.44 \Rightarrow v = 3.8$ (m s^{-1})	**A1**	**2.2a**	**AG –** do not award this mark if using any non-exact values for $\sin\theta / \cos\theta$ (so seeing e.g. $\cos 36.86...$ in their working loses this final **A** mark)	**Must** see either $v^2 = 14.44$ or $v^2 = 36 - 21.56$ or $v^2 = 6^2 - 21.56$ or a correct expression for v before given answer
				[6]			
SC – if m absent throughout (so implying $m = 1$) or m given a numerical value then at most **B0 M1 M1 A1 M1 A0** can be awarded.							

Question	Answer	Marks	AO	Guidance	
	Alternative for final two marks				
	$1.375 = 6t + \frac{1}{2} \times -7.84 \times t^2$ $t = \frac{55}{196}$ (or $t = 1.25$) $v = 6 + (-7.84) \times \frac{55}{196}$	**M1dep***		Use of $s = ut + \frac{1}{2}at^2$ with correct u and s and their negative a $(\neq -9.8)$, **and** selects the smaller of the two positive t values **and** uses this value correctly in $v = u + at$ with correct u and their negative a - **dependent on both previous M marks**	So, using the second positive t (if correct this is 1.25) is **M0**
	$v = 3.8$ (m s^{-1})	**A1**		**AG –** do not award this mark if using any non-exact values for $\sin\theta / \cos\theta$	Must see a correct expression for v before given answer

Question			Answer	Marks	AO	Guidance	
13	(b)		Time from A to B is t_1 where $3.8 = 6 + (-7.84)t_1$	M1	3.4	Use of $v = u + at$ with correct u and v, and their negative a from **(a)** (or any other complete method e.g. $1.375 = \left(\frac{6+3.8}{2}\right)t_1$)	Give **BOD** if $6 = 3.8 + 7.84t_1$ seen but if $v = u + at$ stated before then **M0**
			$[t_1 =] 0.281\,(\text{s})$	A1	1.1	awrt 0.281 or accept 0.28 www. If exact then expect $\frac{55}{196}$	0.2806122...
			[Height of P at B is] 0.825 (m)	B1	3.1b	Must be exact e.g. $\frac{33}{40}$ – allow un-simplified e.g. 1.375×0.6	Allow negative
				M1	3.1b	**M1** for an equation for time t_2 (from B to C) with awrt ± 0.83 for s, $a = \pm g$ and either $u = 3.8 \times \sin\theta$ **or** $u = 3.8 \times \cos\theta$ with $\theta = 37$ (or better) substituted (or corresponding exact value) – condone sign errors	
			$-0.825 = (3.8 \times 0.6)t_2 + 0.5(-g)t_2^2$ or $-0.825 = (3.8 \times \sin 36.86...)t_2 + 0.5(-g)t_2^2$ $(4.9t_2^2 - 2.28t_2 - 0.825 = 0)$	A1	1.1	**A1** for a correct (un-simplified) equation for t_2 - must be using 0.825 but allow $\theta = 37$ or better	For reference: $t_2 = 0.704346599...$
			$T\,[= 0.280... + 0.704...] = 0.985$	A1	2.2a	awrt 0.985 only – **dependent on all previous marks in this part**	0.984958…
				[6]			

APPENDIX

Rules for solving quadratics in questions 2 and 4(b)(i) ONLY

In questions **2** and **4(b)(i)** candidates are required to solve 3 term quadratics (3TQ) using **DR** – therefore we must see a correct, complete method for solving these quadratics – the correct answers do not imply the corresponding M mark, for example in question 2, $9x^2-38x+8=0 \Rightarrow x=4$ or $x=\frac{2}{9}$ is **M0**

Rules for factorising:

$at^2+bt+c \Rightarrow (mt+n)(pt+q)$ where $a=mp$ **and** one of $mq+np=b$ **or** $c=nq$ (so when expanding their factorised expression it must give the correct quadratic term and one other term correct of the preceding 3TQ expression/equation)

e.g. in question **2** (and similarly for question **4(b)(i)**):

$9x^2-38x+8=(x-\frac{2}{9})(x-4)$ is **M0** (but the following **B1** for the correct c.v. of $\frac{2}{9}$ and 4 in qu. 2 can still be awarded as they follow from these two factors)

$9x^2-38x+8=(3x+8)(3x+1)$ is **M1** (when expanded the x^2 and constant terms are correct)

Allow correct part factorisation for their 3TQ expression e.g. if correct 3TQ then in question 2 the expression $9x(x-4)-2(x-4)$ scores **M1**

Rules for the formula:

Must apply the correct formula for their three-term quadratic (no errors even if correct formula is stated) – note that stating the formula (in terms of *a*, *b* and *c*) followed immediately by the corresponding roots is **M0** – we **must** see the formula being applied e.g. $9x^2-38x+8=0 \Rightarrow x=\frac{38\pm\sqrt{38^2-4(9)(8)}}{2(9)}$.

Minimal acceptable working would be $x=\frac{38\pm\sqrt{1156}}{18}$ (so must explicitly see the discriminant) for **M1**

Rules for completing the square – using $9x^2-38x+8=0$ as an example:

The **M1** is not awarded until correctly getting to the stage of $x-\frac{19}{9}=\pm\sqrt{\frac{289}{81}}$ (must include $\pm$ so implying two roots) with no errors (so consistent with applying the formula correctly)

Alternative for 4(b)(ii)

$2\cot^2 2\theta - 9\,\text{cosec}\, 2\theta - 3[=0] \Rightarrow \frac{2}{(2\theta)^2} - \frac{9}{2\theta} - 3\,[=0]$	**M1**	Use of small angle approximations $\sin 2\theta \approx 2\theta$ and $\tan 2\theta \approx 2\theta$ in the given equation from **(a)** to obtain what would be equivalent to a three-term quadratic in θ (allow un-simplified). Allow **M1** only for $\frac{2}{\theta^2} - \frac{9}{\theta} - 3\,[=0]$ (so for using θ instead of 2θ) – allow e.g. x for θ
$6\theta^2 + 9\theta - 1 = 0 \Rightarrow \theta = 0.10(39...)$ so is accurate to 2 dp	**A1**	State 0.10 (or better e.g. 0.1039…) as a decimal following a correct quadratic in θ (no method required for solving quadratic) and comment that this is accurate to 2 dp (as a minimum must mention '2 dp' with the value of 0.10(39…) appearing in this part and 0.10 or 0.101 or 0.100(6789…) appearing in part **(b)(i)**

Additional Guidance for 11(a)

There are many ways that candidates are correctly approaching this part:

For example,

$6(R_D \sin 50) = 3(0.5g \cos 50) - (0.5g \sin 50)$

$6(R_D \sin 50) = 0.5g \times (3 - \tan 50)\cos 50$

$6(R_D \sin 50) = 0.5g \times \sqrt{10}\sin(40 - \arctan(\frac{1}{3}))$

$6(R_D \sin 50) = 0.5g \times \sqrt{10}\cos(140 - \arctan(3))$

$6(R_D \sin 50) = 0.5g \times \sqrt{10}\cos(50 + \arctan(\frac{1}{3}))$

Any of these followed by the correct answer of 1.24 would score full marks (we do not need to see any intermediate working). **If these exact expressions are not seen then sufficient working with values that are correct to at least 3 significant figures should be awarded full marks (and those that have 2 significant figures with sufficient working should get partial credit**), e.g.

- $6(R_D \sin 50) = 9.448977... - 3.7536177...$ so seeing $6(R_D \sin 50) = 9.45 - 3.75$ followed by 1.24 is **M1 A1 A1 A1**
- $6(R_D \sin 50) = 9.4 - 3.8$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.22 not 1.24)
- $6(R_D \sin 50) = 5.7(0)$ is **no marks** (assume working backwards from the **AG** and no indication that the RHS contains a weight component)
- $6(R_D \sin 50) = 0.5g \times \cos 50 \times 1.808246...$ so seeing $6(R_D \sin 50) = 0.5g \times \cos 50 \times 1.81$ followed by 1.24 is **M1 A1 A1 A1**
- $6(R_D \sin 50) = 0.5g \times \cos 50 \times 1.80$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.23 not 1.24)
- $6(R_D \sin 50) = 0.5g \times 0.643 \times 1.81$ followed by 1.24 is **M1 A1 A1 A1**
- $6(R_D \sin 50) = 0.5g \times 0.64 \times 1.8$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.23 not 1.24)
- $6(R_D \sin 50) = 0.5g \times 1.16$ followed by 1.24 is **M1 A1 A0 A0** (assume working backwards from the given answer on the RHS)
- $6(R_D \sin 50) = 0.5g \times \sqrt{10}\sin(40 - 18.434948...)$ so seeing $6(R_D \sin 50) = 0.5g \times 3.16 \times \sin(21.6)$ followed by 1.24 is **M1 A1 A1 A1**
- $6(R_D \sin 50) = 0.5g \times 3.16 \times \sin(22)$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.26 not 1.24)
- $6(R_D \sin 50) = 0.5g \times 3.2 \times \sin(22)$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.28 not 1.24)
- $6(R_D \sin 50) = 0.5g \times \sqrt{10}\cos(68.4349...)$ so seeing $6(R_D \sin 50) = 0.5g \times \sqrt{10} \times \cos(68)$ followed by 1.24 is **M1 A1 A1 A0** (leads to 1.26 not 1.24)

Additional Guidance for 12(c)

The square brackets around the k in the main scheme means that this may be omitted as they could deal with this constant at the end when they form their equation with the 84 (as the k is a constant factor for both expressions for v)

The first **B** mark is for a correct expression/value **for both** $I_1 = [k]\frac{1}{12}\int_0^3 \left(t^2 - 3t\right)\,\mathrm{d}t$ **and** $I_2 = [k]\frac{1}{12}\int_3^6 \left(t^2 - 3t\right)\,\mathrm{d}t$ in any un-simplified form:

so for I_1 any expression that is equivalent to $\pm\frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k]$ is fine (allow either positive or negative due to the fact that this area is below the t-axis) e.g. $I_1 = \pm\frac{1}{12}\left(9 - \frac{27}{2}\right)[k]$ or $\pm\frac{9}{24}[k]$ etc. (ISW once a correct un-simplified form is seen)

For I_2 any expression that is equivalent to $\frac{1}{12}\left(\frac{6^3}{3} - \frac{3(6)^2}{2}\right)[k] - \frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k]$ is fine e.g. $\frac{1}{12}(72 - 54)[k] - \frac{1}{12}(9 - \frac{27}{2})[k]$ or $\left(\frac{3}{2} + \frac{3}{8}\right)[k]$ etc.

If the candidate does not consider these two integrals separately and instead attempts to combine as a single integral (between 0 and 6) then they must consider it correctly (given the applied context) so e.g.

$I_{1,2} = [k]\frac{1}{12}\int_0^6 \left(t^2 - 3t\right)\,\mathrm{d}t = -\frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k] + \frac{1}{12}\left(\frac{6^3}{3} - \frac{3(6)^2}{2}\right)[k] - \frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k]$ scores **B1** but $+\frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k] + \frac{1}{12}\left(\frac{6^3}{3} - \frac{3(6)^2}{2}\right)[k] - \frac{1}{12}\left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)[k]$ is **B0**

The second **B** mark is for a correct expression/value for $I_3 = 54[k]\int_6^9 t^{-2}\,\mathrm{d}t$ in any un-simplified form: e.g. $I_3 = 54\left(-\frac{1}{9} - (-\frac{1}{6})\right)[k]$ or $54\left(-\frac{1}{9} + \frac{1}{6}\right)[k]$ etc. (ISW once a correct un-simplified form is seen)

The third **M** mark is for considering $|I_1| + I_2 + I_3 = 84$ to form a linear equation in k (with the correct number of relevant terms) – this mark is dependent on the first two **M** marks and also they must have taken the modulus or equivalent for their integral between 0 and 3 for this mark e.g. they must have considered $-[k]\frac{1}{12}\int_0^3 \left(t^2 - 3t\right)\,\mathrm{d}t$ **or** $[k]\frac{1}{12}\int_3^0 \left(t^2 - 3t\right)\,\mathrm{d}t$ oe

As this question is detailed reasoning the stages as shown in the MS must all be done to award each corresponding mark. So, an answer of $k = 16$ with no working scores **B1** only.

Alternative Energy Approach to 13(a)

$R(\text{perp. plane}): R = mg\cos 36.86...$ or $mg \times 0.8$	**B1**	Allow using an angle of 37 or better soi (possibly in Work-Energy equation)
$F = 0.25 \times mg\cos 36.86...$ or $0.25 \times mg \times 0.8$	**M1**	Use of $F = 0.25R$ with their R (with $R \neq mg$) – using an angle of 37 or better – allow sin/cos mix R must be a component of weight not mass soi (possibly in W-E equation)
Work done by gravity/PE term = $\pm mg \times (1.375 \times 0.6)$ **or** $\pm mg \times (1.375 \times \sin 36.86...)$ **OR** Work done by friction $\pm 1.375 \times (0.25 \times mg \times 0.8)$ **or** $\pm 1.375 \times (0.25 \times mg \times \cos 36.86...)$	**B1**	Correct expression for either WD by gravity **or** friction – using an angle of 37 or better soi (possibly in W-E equation)
$\frac{1}{2}mv^2 - \frac{1}{2}m \times 6^2 = -mg(1.375 \times 0.6) - 1.375 \times (0.25 \times mg \times 0.8)$	**M1**	Attempt at W-E principle – equation with the correct number of relevant terms, dimensionally correct, allow sign errors and sin/cos mix, **M0** if PE term is $\pm 1.375mg$
	A1	Correct W-E equation
$v^2 = 14.44 \Rightarrow v = 3.8$ (m s^{-1})	**A1**	**AG –** do not award this mark if using any non-exact values for $\sin\theta / \cos\theta$ **Must** see either $v^2 = 14.44$ or $v^2 = 36 - 21.56$ or $v^2 = 6^2 - 21.56$ or a correct expression for v before given answer

SC – if m absent throughout (so implying $m = 1$) or m given a numerical value then at most **B0 M1 B1 M1 A1 A0** can be awarded.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

OCR is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored. © OCR 2024 Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee. Registered in England. Registered office The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA.

Registered company number 3484466. OCR is an exempt charity.

OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.