A-level
FURTHER MATHEMATICS
7367/2

Paper 2

Mark scheme

June 2025

Version: 1.0 Final

256A7367/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Ticks 4th answer.	1.1b	B1	$b = -ac$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles 3rd answer.	2.2a	B1	30 cm^2
	Question total		**1**	

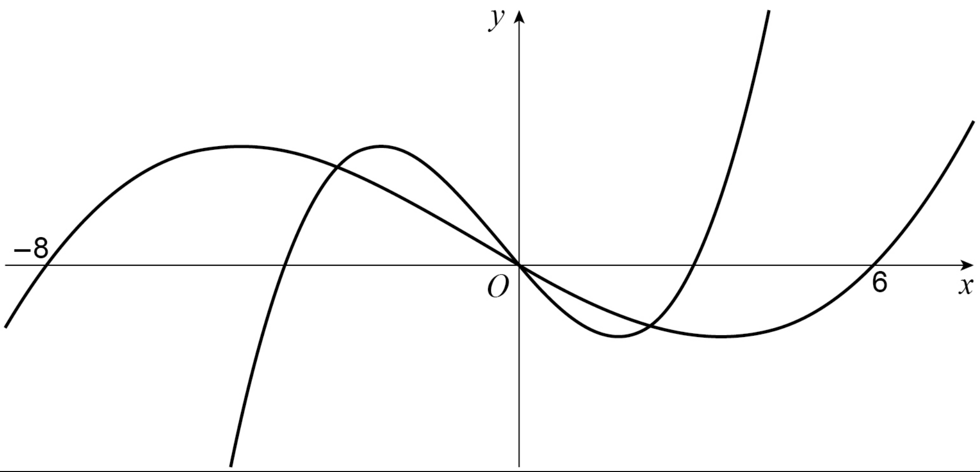
Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles 4th answer.	1.1b	B1	$\dfrac{3}{\sqrt{1-x^2}}$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Ticks 1st answer.	2.2a	B1	$0 \le x \le 1$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Obtains $15 - 4c$	1.1a	M1	$\text{Re}(z) = 15 - 4c$ $15 - 4c = 7$ $c = 2$
	Obtains $c = 2$	1.1b	A1	
	Question total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Substitutes into Euler's formula with 6.069 as the subject (allow one slip).	1.1a	M1	$y_1 = y_0 + h\mathrm{f}(x_0)$ $6.069 = k + \dfrac{0.1(4-3k)}{2k}$ $6.069 = k - 0.15 + \dfrac{0.2}{k}$ $k^2 - 6.219k + 0.2 = 0$ $k = 6.187$ or $k = 0.032$ $k > 1 \Rightarrow k = 6.187$
	Using Euler's formula, forms and obtains a solution to a quadratic equation in k	1.1a	M1	
	Deduces AWRT 6.187 Accept no mention made of rejected root, or root rejected for the wrong reason.	2.2a	A1	
	Question total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Defines elements of **B** (PI) and writes or uses the matrix equation **AB** $= 0$ PI	3.1a	M1	Let $\mathbf{B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\mathbf{AB} = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $= \begin{bmatrix} 4a-2c & 4b-2d \\ -6a+3c & -6b+3d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $4a - 2c = 0 \Rightarrow c = 2a$ $4b - 2d = 0 \Rightarrow d = 2b$ $\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$
	Obtains $c = 2a$ or $d = 2b$ OE PI by correct answer.	1.1a	M1	
	Deduces a matrix of the form $\begin{bmatrix} n & m \\ 2n & 2m \end{bmatrix}$ with at least one of n and m not equal to zero.	2.2a	A1	
	Question total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Draws a cubic graph of the correct shape. Condone any horizontal stretch.	1.1b	B1	See diagram below.
	Draws a cubic graph that crosses x-axis at –8, 0 and 6 Accept no indication of x = 0 at the origin.	1.1b	B1	
	Shows local maximum and minimum points which are approximately the same height as in the original graph.	1.1b	B1	
				
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	Obtains a set of values of the form $\{x : a < x < 2a\}$ or $\{x : 2a < x < a\}$ Condone non-strict inequality. Condone set notation not used.	2.2a	M1	$\{x : 3 < x < 6\}$
	Obtains $3 < x < 6$ Condone set notation not used.	2.2a	A1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Deduces that $\lvert z\rvert = \lvert z+1\rvert$ or $\dfrac{\lvert z\rvert}{\lvert z+1\rvert}=1$	2.2a	B1	Let $z = a + b\mathrm{i}$ $\lvert z\rvert = \lvert z+1\rvert$ $a^2 + b^2 = (a + 1)^2 + b^2$ $a^2 + b^2 = a^2 + 2a + 1 + b^2$ $2a + 1 = 0$ $a = -\dfrac{1}{2}$ $\mathrm{Re}(z) = -\dfrac{1}{2}$
	Obtains and solves an equation in $\mathrm{Re}(z)$	1.1a	M1	
	Obtains $-\dfrac{1}{2}$	1.1b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(b)	Uses the complex cube roots of unity Or Expands and obtains a quadratic equation.	3.1a	M1	$\left(\dfrac{w}{w+1}\right)^3 = 1$ $\dfrac{w}{w+1} = 1,\ \dfrac{w}{w+1} = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}},\ \dfrac{w}{w+1} = \mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}$ $\dfrac{w}{w+1} = 1 \Rightarrow w = w+1$ (impossible) $\dfrac{w}{w+1} = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}} \Rightarrow w = w\left(\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}\right) + \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}$ $w\left(1-\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}\right) = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}$ $w = \dfrac{\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}$ Also $\dfrac{w}{w+1} = \mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}} \Rightarrow w = \dfrac{\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}$ So the only solutions of the equation are $w = \dfrac{\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}$ and $w = \dfrac{\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}$
	Solves an equation in w to obtain at least one correct root.	1.1a	M1	
	Explains why one root is impossible Or Converts at least one of the given solutions into the form a +ib	2.4	E1	
	Completes a reasoned argument to show that the (only) solutions of the equation are $w = \dfrac{\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}}$ and $w = \dfrac{\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}{1-\mathrm{e}^{-\frac{2\pi\mathrm{i}}{3}}}$ May use $\pm$ notation.	2.1	R1	
	Subtotal		**4**	

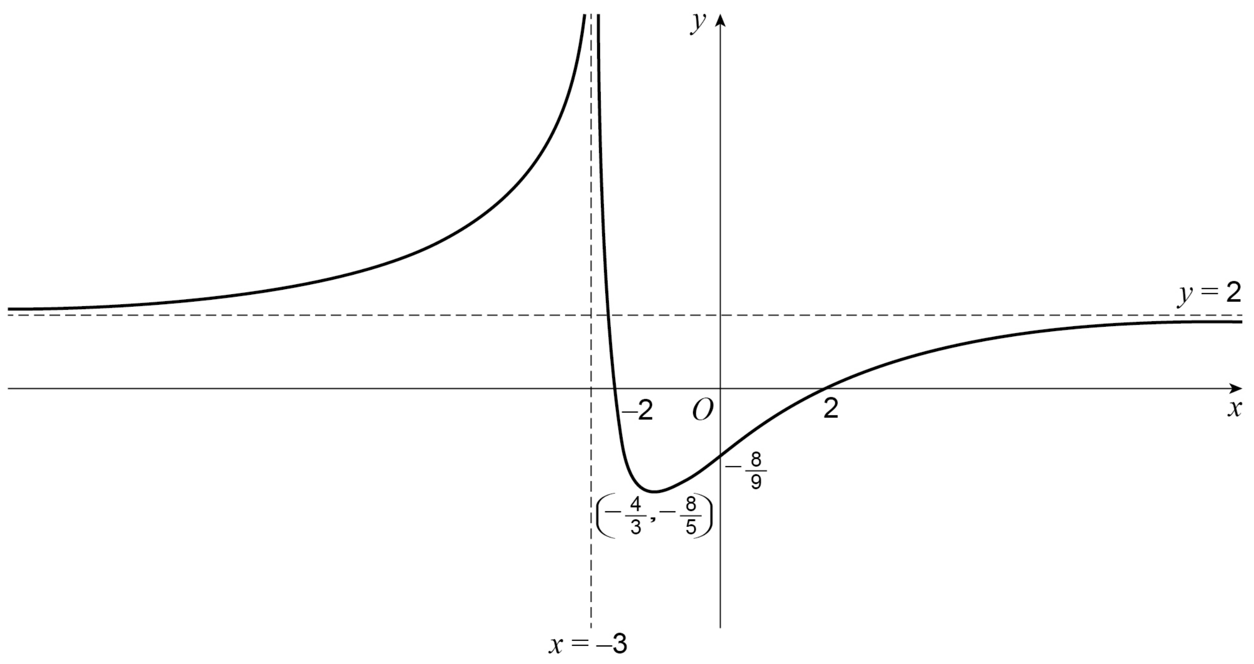
Q	Marking Instructions	AO	Marks	Typical Solution
9(c)	Obtains $-\frac{1}{2}$	2.2a	M1	Let $w=\dfrac{e^{\frac{2\pi i}{3}}}{1-e^{\frac{2\pi i}{3}}}$ Then $\left(\dfrac{w}{w+1}\right)^3=1$ So $\left\|\dfrac{w}{w+1}\right\|=1$ and $\text{Re}(w)=-\dfrac{1}{2}$
	Obtains $-\frac{1}{2}$ from correct reasoning using part **(a)** and/or part **(b)**.	2.1	R1	
	Subtotal		**2**	

	Question total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Obtains $x = -3$ or $y = 2$	1.1a	M1	$x = -3$ $y = 2$
	Obtains $x = -3$ and $y = 2$ and no other asymptotes.	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(b)	Forms a quadratic equation in x involving k or equivalent.	3.1a	M1	Let $y = k$ $k(x^2 + 6x + 9) = 2x^2 - 8$ $(k - 2)x^2 + 6kx + (9k + 8) = 0$ $\Delta \geq 0$ $\therefore (6k)^2 - 4(k-2)(9k+8) \geq 0$ $36k^2 - 36k^2 + 40k + 64 \geq 0$ $40k + 64 \geq 0$ $k \geq -\frac{8}{5}$ The range of f is $\left\{k : k \geq -\frac{8}{5}\right\}$
	Obtains a quadratic equation with k in a coefficient and sets the discriminant to zero, either in an equation or inequality.	1.1a	M1	
	Solves their equation or inequality in k	1.1a	M1	
	Completes a reasoned argument, without using calculus, to state that the range is $\left\{k : k \geq -\frac{8}{5}\right\}$ AG Allow letters other than k used.	2.1	R1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(c)	Obtains a quadratic equation by substituting $k=-\frac{8}{5}$	3.1a	M1	$k=-\frac{8}{5}$ $\frac{-18}{5}x^2+\frac{-48}{5}x+\frac{-32}{5}=0$ $9x^2+24x+16=0$ $(3x+4)^2=0$ $x=-\frac{4}{3}$ Stationary point $=\left(-\frac{4}{3},-\frac{8}{5}\right)$
	Solves their quadratic equation	1.1a	M1	
	Deduces $\left(-\frac{4}{3},-\frac{8}{5}\right)$	2.2a	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(d)	Draws approximately correct shape of the right-hand branch	1.1b	B1	See diagram below.
	Draws both branches of the graph approaching the correct asymptotes.	1.1b	B1	
	Shows at least two correctly labelled axis intercepts.	1.1b	B1	
	Draws completely correct graph including correctly labelled asymptotes, a stationary point in the third quadrant, and all three correctly labelled axis intercepts. Condone no labelling of stationary point.	1.1b	B1	
	 			
	Subtotal		**4**	

	Question total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)	Deduces a vector from *A* to a general point on the line.	2.2a	M1	At the closest point, $\overrightarrow{AP} \bullet \underline{l} = 0$ $\overrightarrow{AP} = \begin{bmatrix} 4+2\lambda \\ 3-\lambda \\ 17+3\lambda \end{bmatrix}$ $\begin{bmatrix} 4+2\lambda \\ 3-\lambda \\ 17+3\lambda \end{bmatrix} \bullet \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = 0$ $8+4\lambda-3+\lambda+51+9\lambda=0$ $56+14\lambda=0$ $\lambda=-4$ $r = \begin{bmatrix} 5 \\ 2 \\ 11 \end{bmatrix} + (-4)\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -1 \end{bmatrix}$ The closest point is (–3, 6, –1)
	Selects a suitable method, for example: Uses the scalar product Or Obtains an expression for $\left\lvert\overrightarrow{AP}\right\rvert$ or $\left\lvert\overrightarrow{AP}\right\rvert^2$	3.1a	M1	
	Forms and solves an equation in λ	1.1a	M1	
	Substitutes their value of λ into the vector equation of *L*	1.1a	M1	
	Obtains (–3, 6, –1) Condone position vector.	1.1b	A1	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(b)	Obtains two vectors in the plane.	1.1a	M1	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} -12 \\ 1 \\ 5 \end{bmatrix}$ $-12 \times 4 + 1 \times 0 + 5 \times 1 = -43$ $-12x + y + 5z = -43$
	Deduces a normal vector from their two vectors Or Obtains a vector equation of the plane in the form $\mathbf{r} = \mathbf{a} + \lambda \overrightarrow{AB} + \mu \overrightarrow{BC}$ or similar.	2.2a	M1	
	Uses the position vector of *A*, *B* or *C* to obtain the constant term Or Uses their vector equation to obtain and solve three simultaneous equations	1.1a	M1	
	Obtains a correct Cartesian equation.	1.1b	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(c)	Uses the equation of the line *L* and their equation of the plane Π to form an equation in λ	3.1a	M1	$-12x + y + 5z = -43$ $-12(5 + 2\lambda) + (2 - \lambda) + 5(11 + 3\lambda) = -43$ $-60 - 24\lambda + 2 - \lambda + 55 + 15\lambda + 43 = 0$ $\lambda = 4$ The point is (13, –2, 23)
	Solves equation in λ	1.1a	M1	
	Obtains (13, –2, 23) Condone position vector.	1.1b	A1	
	Subtotal		**3**	

	Question total		**12**	

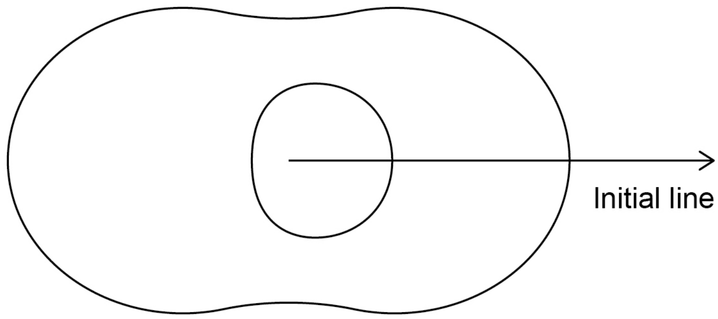
Q	Marking Instructions	AO	Marks	Typical Solution
12	Completes the square to obtain $(x+p)^2 - q^2$	3.1a	M1	$\int_1^5 \frac{1}{\sqrt{x^2+6x+5}}\,\mathrm{d}x = \int_1^5 \frac{1}{\sqrt{(x+3)^2-2^2}}\,\mathrm{d}x$ $= \left[\cosh^{-1}\left(\frac{x+3}{2}\right)\right]_1^5$ $= \cosh^{-1}4 - \cosh^{-1}2$ $= \ln(4+\sqrt{15}) - \ln(2+\sqrt{3})$ $= \ln\left(\frac{4+\sqrt{15}}{2+\sqrt{3}}\right)$ $= \ln\left(\left(\frac{4+\sqrt{15}}{2+\sqrt{3}}\right)\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)\right)$ $= \ln\left(\frac{8-4\sqrt{3}+2\sqrt{15}-\sqrt{45}}{4-3}\right)$ $= \ln(8-4\sqrt{3}-3\sqrt{5}+2\sqrt{15})$
	Integrates to obtain $\cosh^{-1}\left(\frac{x+p}{q}\right)$ or $\ln(x+p+\sqrt{(x+p)^2-q^2})$	1.1b	A1F	
	Substitutes the limits and deduces $\ln(4+\sqrt{15}) - \ln(2+\sqrt{3})$ or $\ln(8+\sqrt{60}) - \ln(4+\sqrt{12})$	2.2a	A1	
	Uses laws of logs to simplify result of their integration.	1.1a	M1	
	Completes a reasoned argument to obtain $\ln(8-4\sqrt{3}-3\sqrt{5}+2\sqrt{15})$	2.1	R1	
	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Forms a matrix equation Or Forms equations in r and θ PI by $r=2$ or $\theta=\frac{\pi}{3}$	3.1a	M1	$\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}=\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ $r\cos\theta=1$ $r\sin\theta=\sqrt{3}$ $r=2$ $\theta=\frac{\pi}{3}$
	Deduces $r=2$ or $\theta=\frac{\pi}{3}$	2.2a	A1	
	Obtains $r=2$ and $\theta=\frac{\pi}{3}$	1.1b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(b)	Uses matrix multiplication to deduce $u=x-\sqrt{3}y$ and $v=\sqrt{3}x+y$	2.2a	B1	$\begin{bmatrix} u \\ v \end{bmatrix}=\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} x-\sqrt{3}y \\ \sqrt{3}x+y \end{bmatrix}$ $u=x-\sqrt{3}y,\ v=\sqrt{3}x+y$ $r\,\mathrm{e}^{\mathrm{i}\theta}(x+\mathrm{i}y)=2\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}(x+\mathrm{i}y)$ $=2\left(\cos\frac{\pi}{3}+\mathrm{i}\sin\frac{\pi}{3}\right)(x+\mathrm{i}y)$ $=(1+\mathrm{i}\sqrt{3})(x+\mathrm{i}y)$ $=x-\sqrt{3}y+\mathrm{i}(\sqrt{3}x+y)$ $=u+\mathrm{i}v$
	Uses their values of r and θ to obtain an expression for $r\,\mathrm{e}^{\mathrm{i}\theta}(x+\mathrm{i}y)$ with no trigonometric or exponential terms.	1.1a	M1	
	Uses correct reasoning to obtain $x-\sqrt{3}y+\mathrm{i}(\sqrt{3}x+y)$ and to conclude that $r\,\mathrm{e}^{\mathrm{i}\theta}(x+\mathrm{i}y)=u+\mathrm{i}v$ or $2\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}(x+\mathrm{i}y)=u+\mathrm{i}v$	2.1	R1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(c)	Deduces $\left(2e^{\frac{i\pi}{3}}\right)^8(x+iy)=2i$ OE	2.2a	B1	$\mathbf{M}^8\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\2\end{bmatrix}\Rightarrow\left(2e^{\frac{i\pi}{3}}\right)^8(x+iy)=2i$ $x+iy=\left(2e^{\frac{i\pi}{3}}\right)^{-8}(2i)$ $=2^{-8}e^{\frac{-8i\pi}{3}}(2i)$ $=2^{-8}\left(\cos\left(\frac{-8\pi}{3}\right)+i\sin\left(\frac{-8\pi}{3}\right)\right)(2i)$ $=\frac{1}{256}\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)(2i)$ $=\frac{1}{256}(\sqrt{3}-i)$ $x=\frac{\sqrt{3}}{256},y=\frac{-1}{256}$
	Forms an equation including 2^8 or 2^{-8}, and $e^{\pm\frac{8\pi i}{3}}$ or $e^{\pm\frac{2\pi i}{3}}$ OE Or Obtains the modulus and argument of $x+iy$	3.1a	M1	
	Obtains values of x and y, including no trigonometric or exponential terms, from an equation including $e^{\pm\frac{8\pi i}{3}}$ or $e^{\pm\frac{2\pi i}{3}}$	1.1a	M1	
	Obtains $x=\frac{\sqrt{3}}{256},y=\frac{-1}{256}$	1.1b	A1	
	Subtotal		**4**	

	Question total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Draws closed curve, approximately symmetrical about the initial line, completely inside the existing curve.	1.1b	B1	See diagram below.
	Draws closed curve with pole inside their shape, with more of the curve to the right.	1.1b	B1	
				
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(b)	Deduces a correct expression for the total area (PI). Condone omission of ½	2.2a	M1	$A = 2\times\frac{1}{2}\times\int_0^{\pi}(6+2\cos 2\theta)^2\,\mathrm{d}\theta$ $-2\times\frac{1}{2}\times\int_0^{\pi}(2+\cos\theta)^2\,\mathrm{d}\theta$ $=\int_0^{\pi}\left(36+24\cos 2\theta+4\cos^2 2\theta\right)\mathrm{d}\theta$ $-\int_0^{\pi}(4+4\cos\theta+\cos^2\theta)\mathrm{d}\theta$ $=\int_0^{\pi}\left(36+24\cos 2\theta+2+2\cos 4\theta\right)\mathrm{d}\theta$ $-\int_0^{\pi}(4+4\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta)\mathrm{d}\theta$ $=\left[38\theta+12\sin 2\theta+\frac{1}{2}\sin 4\theta\right]_0^{\pi}$ $-\left[\frac{9}{2}\theta+4\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{\pi}$ $=\left(38\pi-0\right)-\left(\frac{9}{2}\pi-0\right)=\frac{67\pi}{2}$ $V=0.3\times\frac{67\pi}{2}=\frac{201\pi}{20}$ $V = 31.57$ m^3
	Uses a trig identity to correctly transform $4\cos^2 2\theta$ or $\cos^2\theta$	3.1a	M1	
	Integrates $(6+2\cos 2\theta)^2$ or $(2+\cos\theta)^2$ correctly.	1.1b	A1	
	Substitutes limits correctly for at least one integral.	1.1a	M1	
	Obtains $\frac{67\pi}{2}$ from correct working.	1.1b	A1	
	Obtains AWRT 31.57 m^3 Must include units.	3.2a	A1	
	Subtotal		**6**	

	Question total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
15	Expresses the integrand as partial fractions.	3.1a	M1	$\dfrac{1}{(x+1)(2x+3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{2x+3}$ $1 \equiv A(2x+3) + B(x+1)$
	Obtains a correct expression for the integrand as partial fractions.	1.1b	A1	$x = -1:\ 1 = A$ $x = 0:\ \ 1 = 3A + B \Rightarrow B = -2$
	Integrates their expression correctly to obtain an expression involving logs.	1.1a	M1	$\displaystyle\int_5^{\infty} \frac{1}{(x+1)(2x+3)}\,\mathrm{d}x$ $\displaystyle= \int_5^{\infty} \left(\frac{1}{x+1} - \frac{2}{2x+3}\right)\mathrm{d}x$
	Clearly shows the limiting process by setting the upper limit to N or equivalent (not x) and taking the limit at some stage.	2.4	M1	$\displaystyle= \lim_{N\to\infty} \int_5^{N} \left(\frac{1}{x+1} - \frac{2}{2x+3}\right)\mathrm{d}x$ $\displaystyle= \lim_{N\to\infty} \left[\ln(x+1) - \ln(2x+3)\right]_5^N$
	Deduces $\displaystyle\lim_{N\to\infty}\left(\ln\left(\frac{N+1}{2N+3}\right)\right) = \ln\left(\frac{1}{2}\right)$ Or $\displaystyle\lim_{N\to\infty}\left(\ln\left(\frac{13(N+1)}{6(2N+3)}\right)\right) = \ln\left(\frac{13}{12}\right)$ OE	2.2a	A1	$\displaystyle= \lim_{N\to\infty}\left[\ln\left(\frac{x+1}{2x+3}\right)\right]_5^N$ $\displaystyle= \lim_{N\to\infty}\left(\ln\left(\frac{N+1}{2N+3}\right) - \ln\left(\frac{6}{13}\right)\right)$ $\displaystyle= \lim_{N\to\infty}\left(\ln\left(\frac{1+\frac{1}{N}}{2+\frac{3}{N}}\right) - \ln\left(\frac{6}{13}\right)\right)$ $\displaystyle= \ln\left(\frac{1}{2}\right) - \ln\left(\frac{6}{13}\right)$
	Completes a rigorous argument to obtain the correct result. Must include $\ln\left(\dfrac{1+\frac{1}{N}}{2+\frac{3}{N}}\right)$ oe	2.1	R1	$\displaystyle= \ln\left(\frac{13}{12}\right)$
	Question total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
16	Obtains a correct expression for the area. Condone omission of "2 ×"	3.1a	B1	$A = 2\times\int_{0.5}^{1.5}\sqrt{\frac{0.27}{2x-x^2}}\,\mathrm{d}x$ $A = (2\times\sqrt{0.27})\int_{0.5}^{1.5}\frac{1}{\sqrt{1-(x-1)^2}}\,\mathrm{d}x$ $=\left(\frac{3\sqrt{3}}{5}\right)\left[\sin^{-1}(x-1)\right]_{0.5}^{1.5}$ $=\left(\frac{3\sqrt{3}}{5}\right)(\sin^{-1}(0.5)-\sin^{-1}(-0.5))$ $=\left(\frac{3\sqrt{3}}{5}\right)\left(\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)\right)$ $=\left(\frac{3\sqrt{3}}{5}\right)\left(\frac{\pi}{3}\right)=\frac{\pi\sqrt{3}}{5}$ m^2
	Completes the square for the denominator of the integrand to obtain $1-(x\pm1)^2$	3.1a	M1	
	Integrates to obtain an inverse sine function.	1.1a	M1	
	Obtains $k\sin^{-1}(x-1)$ and substitutes in the limits.	1.1a	M1	
	Completes a reasoned argument to obtain $\frac{\pi\sqrt{3}}{5}$ Condone missing units.	2.1	R1	
	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Obtains $\frac{dy}{dt} = \pm k(y - u)$ OE	3.3	M1	$\frac{dy}{dt} = k(y - u)$ $\frac{dy}{dt} = k(y - 18 + 2t)$ Initially $-0.8 = k(38 - 18) = 20k$ $k = -0.04$ $\frac{dy}{dt} = -0.04(y - 18 + 2t)$ $= -0.04y + 0.72 - 0.08t$ $\frac{dy}{dt} + 0.04y = 0.72 - 0.08t$
	Substitutes initial conditions into their differential equation to obtain $k = \pm 0.04$	3.4	M1	
	Completes a reasoned argument to obtain $\frac{dy}{dt} + 0.04y = 0.72 - 0.08t$	2.1	R1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(b)	Selects a suitable method eg obtains an integrating factor or Obtains a complementary function.	3.1a	M1	$\frac{dy}{dt}+0.04y=0.72-0.08t$ $\text{IF}=e^{\int 0.04\,dt}=e^{0.04t}$ $e^{0.04t}\left(\frac{dy}{dt}+0.04y\right)=e^{0.04t}\left(0.72-0.08t\right)$ $\frac{d}{dt}\left(ye^{0.04t}\right)=0.72e^{0.04t}-0.08te^{0.04t}$ $ye^{0.04t}=0.72\int e^{0.04t}\,dt-0.08\int te^{0.04t}dt$ $u=t \quad v'=e^{0.04t}$ $u'=1 \quad v=25e^{0.04t}$ $ye^{0.04t}=0.72\left(25e^{0.04t}\right)-0.08\left(25te^{0.04t}\right)$ $+0.08\left(25\right)\int e^{0.04t}dt$ $=18e^{0.04t}-2te^{0.04t}+50e^{0.04t}+c$ $y=68-2t+ce^{-0.04t}$ When $t=0$, $y=38\Rightarrow c=-30$ $y=68-2t-30e^{-0.04t}$
	Obtains correct integrating factor or Obtains correct complementary function.	1.1b	A1	
	Multiplies equation by their integrating factor or Uses correct form of particular integral.	1.1a	M1	
	Uses integration by parts on RHS or Equates coefficients of t and equates constant coefficients.	3.1a	M1	
	Obtains $18e^{0.04t}-2te^{0.04t}+50e^{0.04t}$ or Obtains correct particular integral $y=68-2t$	1.1b	A1	
	Uses initial conditions to obtain constant of integration or coefficient of exponential term.	3.1b	M1	
	Obtains $y=68-2t-30e^{-0.04t}$	1.1b	A1	
	Subtotal		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(c)	Obtains $y - u = 28$ OE	3.3	M1	$y - u = 28$ $68 - 2t - 30e^{-0.04t} - (18 - 2t) = 28$ $22 = 30e^{-0.04t}$ $e^{-0.04t} = \dfrac{22}{30}$ $t = 7.75387$ 7 minutes 45 seconds
	Forms a correct equation from their y, which is in the form $y = Ae^{\alpha t} + B + Ct$	1.1b	A1F	
	Solves their exponential equation.	3.1a	M1	
	Obtains 7 minutes 45 seconds. Condone 7 minutes 46 seconds.	3.2a	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(d)	States a limitation of the model, with reference to the fact that the model implies that the temperature of the freezer or the sample could decrease indefinitely.	3.5b	E1	The model would not work for large values of t since the temperature of the freezer would not decrease indefinitely.
	Subtotal		**1**	

	Question total		**15**	

	Paper Total		**100**	