AQA
Questions matter

A Level
Further Mathematics
7367/1

Report on the examination

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Introduction

This paper had a good mix of questions from many topics and at all levels of difficulty. This allowed all students to demonstrate their knowledge of basic skills across the specification, whilst allowing the stronger students to demonstrate their deeper understanding of the topics being examined.

Question 1

A straightforward multiple-choice question that the overwhelming majority of students got correct.

Question 2

Another straightforward multiple-choice question that the overwhelming majority of students also got correct.

Question 3

Less students got this multiple-choice question correct than on the first two question, but still a large majority did get it correct.

Question 4

This multiple-choice question threw a sizeable minority of students who used trigonometric identities rather than hyperbolic ones.

Question 5

Question 6 was done well, with only a few students incorrectly thinking that $\arg\left(\frac{z_1}{z_2}\right) = \frac{\arg(z_1)}{\arg(z_2)}$. Most commonly, marks were lost when students struggled to work out $\arg(-2-4\text{i})$ from first principles. Those students who realised that, as the question said "Find $\arg\left(\frac{z_1}{z_2}\right)$ ", they did not need to show detailed working out, they just used their calculators in complex number mode to look up $\arg(-2-4\text{i})$. Virtually all of these students scored full marks.

Question 6

Question 6 was done extremely well. Very few students used 6 ordinates, in which case they did not gain any credit. Most lost marks were for incorrect calculator use after the correct Simpson's Rule was written down.

Question 7

In question 7 students chose one of two distinct approaches to the question:

Approach 1 (as per the typical solution) : they did the variable transformation $z=\frac{w+1}{2}$ as indicated by the new root structure in the question, substituted this into the original equation, expanded and then simplified. This got the new equation relatively quickly. The main error here was getting the wrong constant term.

Approach 2 : they formed Vieta's equations for the new roots in terms of the old roots. This method for polynomials of degree three or higher involves a lot more algebraic manipulation. The mark scheme only awarded credit when the new expressions are written in terms of fully factorised expressions in terms of the old equations. For example, leaving the first expression as $2\alpha+2\beta+2\gamma-3$ with no further working does not gain any credit as it has to be $2(\alpha+\beta+\gamma)-3$ to gain credit.

In general, those students who used approach 1 did much better than students who used approach 2. Teachers are urged to make sure students are aware of this.

Question 8

Question 8 was done very well, with only a few students getting the signs wrong in the transformation to get the equation for C2. Once this hurdle was overcome, nearly all got the correct result, with only a tiny percentage forgetting to write their values of x in coordinate form.

Question 9

This turned out to be a very challenging question. Only the strongest students were able to work their way through it to a successful conclusion. Many tried to work with components and this led to going round in circles for a while before stopping. Some simply wrote down the given general information given about the scalar product and the modulus of the cross product, $-\sqrt{3}=|\mathbf{a}||\mathbf{b}|\cos\theta$ and $12=|\mathbf{a}||\mathbf{b}|\sin\theta$. This sensible approach gained them half the marks for the question. The final group of students then used the last piece of information, $|\mathbf{a}|=7$, and substituted it in. They then realised that this unlocked the rest of the question for them. Some of these students chose to work out the value of the angle, but many of these did not realise that it was obtuse and ran into some issues. The strongest students of all realised that using $\sin^2\theta+\cos^2\theta=1$ made the question drop out very quickly and efficiently.

Question 10

Question 10 was a nice twist on the traditional proof by induction question. Traditionally, properly showing the result is true for $n=1$, focused on in part **(a)**, and putting together the final statement, focused on in part **(b)**, have been weakness for students.

Part **(a)(i)** was done extremely well, with marks mainly being lost by students being too vague for it to be clear that they were referring to $n=1$, or by only stating "base case" or "$n=1$" without any indication as to what the issue was.

Part **(a)(ii)** highlighted a major issue that students have. In proving the $n=1$ case they need to fully show it twice. Here, once for the definition and once for the formula. So writing $2\times1=2=1\times(1+1)$ is not enough. The typical solution in the mark scheme is the type of thing that is required.

Part **(b)(i)** was generally correct.

Marks were lost in Part **(b)(ii)** when students said that both $n=k$ and $n=k+1$ were assumed, clarity of explanation was the key here.

Question 11

In part **(a)** students quickly worked out $\mathrm{f}'(x)$ and then used either the quotient rule or the product rule to workout $\mathrm{f}''(x)$. Unsurprisingly it was in the workings for $\mathrm{f}''(x)$ that student errors tended to creep in. Those students choosing to use the product rule on the whole made less errors with the given answer falling out relatively easily.

In part **(b)** students had to use the work from part **(a)**, as per the hence, and find $\mathrm{f}'''(0)$. Calculating $\mathrm{f}(x), \mathrm{f}'(x)$ and $\mathrm{f}''(x)$ was relatively straightforward. Calculating $\mathrm{f}'''(x)$ to obtain $\mathrm{f}'''(0)$ was less so. Some students though realised they could use their calculators to obtain $\mathrm{f}'''(0)$ and they did this with great success. A few students ignored the hence statement and tried using the expansion for e^x in $\mathrm{f}(x)$. This was not a successful approach, the hence statement here was designed to guide them away from attempting to do this.

Question 12

In question 12 part **(a)** was generally done well. The most common slip was to write the characteristic equation as $\begin{vmatrix} 19-\lambda & 3 \\ 3 & 11-\lambda \end{vmatrix}=0$ leading to $\lambda=20$ and $\lambda=10$. Students need to be careful when there is a fraction outside of the matrix. Another common error was not correctly extracting the eigenvector from the Cartesian equation of the eigenvector, switching the values of x and y was the usual slip here.

Part **(b)** was very challenging with students either unsure about what they were being asked to do or being so vague that they did not have a proper explanation. Strong students were able to show their deeper understanding of the topic here.

Finding the matrices $\mathbf{U}, \mathbf{D}$ and $\mathbf{U}^{-1}$ continues to be a skill that many can show their knowledge on. Part **(c)** was fully follow through so students could restart here if they had not completed part **(a)**.

Following on from part **(c)**, part **(d)** was also very accessible to students. With only a few students ignoring “Hence” and using successive multiplication on their calculators to gain no credit.

Question 13

This question was done extremely well by students of all abilities. Teachers had obviously prepared their students effectively for this topic and that is to their credit. The field was split quite evenly between those who used Vieta’s equations and those who factorised out the quadratic equation with complex roots. Neither method seemed to be more successful than the other.

Question 14

Part **(a)** saw a marked improvement since the last time this sort of question was asked. There was a marked reduction in the number of students finishing the proof with statements such as $y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$, though some still did. A sizeable minority of student though did not start with the exponential definitions of sinh$x$ and cosh$x$, as instructed in the question. Instead, they started with one of the exponential definitions of coth$x$. This prevented them from achieving the final mark of this section. Many of the students who did this were also writing $y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ at the end as well.

Everyone was back in the game in part **(b)** and many students who did not complete part **(a)** scored well in this part. Surprisingly the biggest loss of marks was errors in doing basic logarithmic manipulation. Some students went the route of writing $x = \coth(-\ln 5)$ and then substituting $-\ln 5$ into the exponential definition of coth. As this was not hence, they gained no credit for this approach. It would be worth reinforcing with students what hence at the beginning of a question is instructing them to do.

Question 15

Students found the topic in question 15 extremely challenging.

In part **(a)** it was clear that a number of students did not know how to tackle this question. Those who did generally did well, though some forgot to eliminate to get two equations in the same variables.

Part **(b)** was beyond all but the strongest students, who presented some very nice working. Please note that in questions which ask students to “Fully justify your answer”, it is not acceptable to use a graphical calculator to solve the 3x3 simultaneous equation to get the parametric form of the line of intersection and then extract out the vector equation of the line. This approach (used by about 5% of students) gained no credit.

Question 16

Most students were able to set up the correct formula for the surface area. However, only the strongest students were able to continue the working to get the printed answer. Some students unfortunately lost the last mark, as they did not show a greater degree of accuracy before writing down the final (printed) result. This is a standard requirement across both A level Further Mathematics and A level Mathematics. Students need reminding about this as it occurs frequently. The use of calculators to integrate from various points in the solution was also in evidence, this approach did not gain credit.

Question 17

Whilst students have got noticeably better at setting up a second order differential equations from elastic string scenarios, it is still a place where the strongest of them can show their advanced skills. This was clearly shown in the responses to part **(a)**.

In part **(b)** all students could start from the given differential equation in part **(a)**. There was a large spread of solutions showing that the question was accessible to all whilst allowing the stronger students to show their deep understanding of the topic. A common slip was thinking that $x = -0.5$ when $t = 0$.
An unusual error that occurred a few times was, after getting the correct complex solutions of the Auxiliary Equation, going on to write the general solution as $e^{\frac{-2\sqrt{5}}{3}}\left(\cos\left(\frac{10}{3}t\right)+i\sin\left(\frac{10}{3}t\right)\right)$, which is an issue to lookout for when teaching this topic!

Question 18

Part **(a)** was generally done very well. With most of the lost marks coming from forgetting to include “<0”.
Those who used exponential form (rather than a quadratic in sinh x) did not do as well.

As expected, a number of students appeared to run out of time before getting to part **(b)**. Of those who did attempt it, those who tried to use the exponential form of $\tanh\left(\frac{1}{2}x\right)$ generally got nowhere with it. Those who took the route indicated in the typical solution generally got at least half marks. The step to $\frac{1}{2\sinh\left(\frac{1}{2}x\right)\cosh\left(\frac{1}{2}x\right)}$ proved to be surprisingly challenging due to the necessity to deal with two composite functions. This allowed the stronger students to demonstrate their skills. Only a very few students forgot to include $f'(x) =$ for the last mark.

Some students in part **(c)** appeared to run out of time so did not attempt this question.
This question allowed many students to show their skills, with about half making very good attempts indeed. Common slips were getting 9/2 wrong in A_1 and forgetting the 15 when integrating in A_2.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.