



Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level

FURTHER MATHEMATICS

Paper 1

Thursday 22 May 2025

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	



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Answer **all** questions in the spaces provided.

- 1 The equation $x^2 + 4x + c = 0$ has non-real roots.

Find the range of possible values of c

Circle your answer.

[1 mark]

$c < 4$

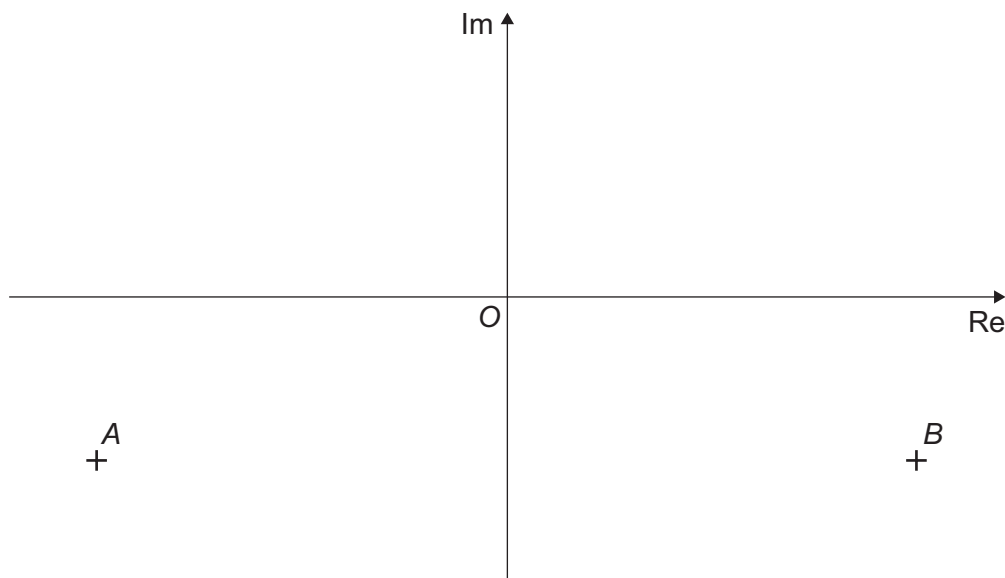
$c \leq 4$

$c \geq 4$

$c > 4$

- 2 The point A on the Argand diagram represents the complex number z

The point B is the reflection in the imaginary axis of the point A



Which complex number does the point B represent?

Circle your answer.

[1 mark]

$-z$

z^*

$-z^*$

$\frac{1}{z}$



- 3 Which **one** of the following expressions **cannot** be evaluated by l'Hôpital's rule?

Circle your answer.

[1 mark]

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt{x}}{\cos x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{x^2}{\sin x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{\tanh x}{2x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{\ln(1+x)} \right)$$

- 4 Which **one** of the following statements is always correct?

Tick (✓) **one** box.

[1 mark]

$$\sinh^2 x = \frac{1}{2}(1 - \cosh 2x)$$

☐

$$\operatorname{sech}^2 x + \tanh^2 x = 1$$

☐

$$\operatorname{cosech}^2 x + \coth^2 x = 1$$

☐

$$\cosh^2 x = \frac{1}{2}(\cosh 2x - 1)$$

☐

Turn over for the next question

Turn over ►



5 The complex number z_1 has argument $\tan^{-1}\left(\frac{1}{5}\right)$

The complex number $z_2 = -2 - 4i$

Find $\arg\left(\frac{z_1}{z_2}\right)$

Give your answer as a number in the range $-\pi < \alpha < \pi$, to **two** decimal places.

[3 marks]



- 6 Use Simpson's rule with 5 ordinates to find an approximation to

$$\int_0^2 \frac{1}{\sqrt{1+x^4}} dx$$

Give your answer to **three** decimal places.

[3 marks]

Turn over for the next question

Turn over ►



7 The cubic equation

$$5z^3 + 4z^2 - z + 3 = 0$$

has roots α , β and γ

Find an equation, with integer coefficients, that has roots $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$

[4 marks]



8

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

The curve C_1 is translated by the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ to give the curve C_2

Find the coordinates of the points where the curve C_2 intersects the x -axis.

[3 marks]

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[4 marks]

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10 Astrid is solving this mathematics problem:

The series S_n is defined by

$$S_n = 2 + 4 + 6 + \dots + 2n \quad (n \in \mathbb{Z}, n \geq 1)$$

Prove by induction that

$$S_n = n(n+1)$$

Astrid's solution is as follows:

Assume the result is true for $n = k$

Then

$$S_k = k(k+1)$$

$$S_{k+1} = S_k + 2(k+1)$$

$$S_{k+1} = k(k+1) + 2(k+1)$$

$$S_{k+1} = (k+2)(k+1)$$

$$S_{k+1} = (k+1)((k+1)+1)$$

So the result is also true for $n = k+1$

The result is true for $n = 1$

It is true for $n = k$, and also true for $n = k+1$

Hence, by induction $S_n = n(n+1)$ for all integers $n \geq 1$

10 (a) (i) Chloe says that Astrid missed out an essential part of the proof, which could have been written at the start.

Explain what Astrid missed out.

[1 mark]



10 (a) (ii) Write down the working that Astrid missed out.

[1 mark]

10 (b) (i) One statement in the last three lines of Astrid's solution is written incorrectly.

Which statement is written incorrectly?

Tick (✓) **one** box.

[1 mark]

The result is true for $n = 1$

☐

It is true for $n = k$, and also true for $n = k + 1$

☐

Hence, by induction $S_n = n(n+1)$ for all integers $n \geq 1$

☐

10 (b) (ii) Write out a correct statement which should replace the incorrect statement identified in part (b)(i)

[1 mark]

Turn over for the next question

Turn over ►



11

$$f(x) = \frac{1}{1 + e^x} \quad (x \in \mathbb{R})$$

11 (a)

$$f''(x) = \frac{-e^x + e^{2x}}{(1+e^x)^3}$$

[3 marks]

[illegible]

[4 marks]

[illegible]

12 (a)

[5 marks]

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12 (b)

[2 marks]



12 (c) Find matrices $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{U}^{-1}$, such that $\mathbf{D}$ is diagonal and $\mathbf{M} = \mathbf{U}\mathbf{D}\mathbf{U}^{-1}$

[3 marks]

12(d) Hence, find the matrix $\mathbf{L}$ such that $\mathbf{M}^n \rightarrow \mathbf{L}$ as $n \rightarrow \infty$

[3 marks]

Turn over for the next question

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13 The function f is defined by

$$f(z) = 4z^3 + rz^2 + 92z + s$$

where r and s are real numbers.

One of the roots of the equation $f(z) = 0$ is $-4 + 3i$

13 (a) Find the other two roots of the equation $f(z) = 0$

[4 marks]



13 (b) Find the value of r and the value of s

[3 marks]

Turn over for the next question

Turn over ►



$$\coth^{-1}(x) = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right)$$
[illegible]

14 (b) Hence, solve the equation

$$\coth^{-1}(x) = -\ln 5$$

Give your answer in an exact form.

[3 marks]

Turn over for the next question

Turn over ►



$$\begin{array}{rcl} x & + & 2y - z = 9 \\ x & - & 3y + 3z = t \\ 3x & + & y + z = 4t \end{array}$$

The planes meet along a line of intersection.

[4 marks]

[illegible]

Fully justify your answer.

[5 marks]

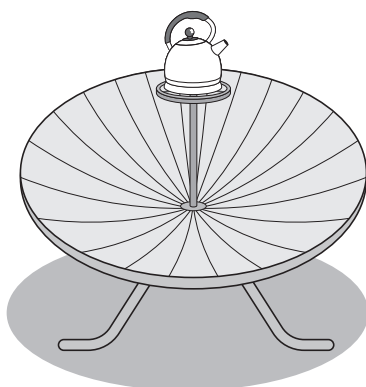
[illegible]

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16



The shape of the internal surface of the dish is formed by rotating the part of the parabola $y^2 = 0.8x$ between $x=0$ and $x=0.25$ through 2π radians about the x -axis, where x and y are measured in metres.

Use integration to show that the internal surface area of the dish, to **three** decimal places, is 0.796 square metres.

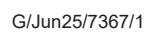
Fully justify your answer.

[7 marks]

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[illegible]

Turn over ►



17 In this question use $g = 10 \text{ m s}^{-2}$

A particle P of mass 0.6 kg is attached to one end of each of two light elastic strings, AP and BP

The other ends of the strings, A and B , are attached to fixed points which are 7 metres apart, with A vertically above B

The natural length of the string AP is 2 metres .

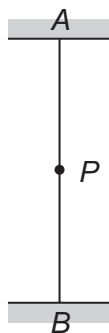
When the extension of the string AP is $e \text{ metres}$, the tension in the string AP is $5e \text{ newtons}$.

The natural length of the string BP is 3 metres .

When the extension of the string BP is $e \text{ metres}$, the tension in the string BP is $3e \text{ newtons}$.

The whole system is in a large tub of oil.

The diagram shows the particle P , the strings and the points A and B



The particle P is held at the point between A and B which is 0.5 metres vertically below its equilibrium position.

The particle is then released from rest.

During the subsequent motion the oil causes a resistive force of magnitude $\frac{4}{\sqrt{5}}v \text{ newtons}$ to act on the particle, where $v \text{ m s}^{-1}$ is the speed of the particle.

At time $t \text{ seconds}$ after P is released, its displacement towards B from its equilibrium position is $x \text{ metres}$.



$$0.6 \frac{d^2x}{dt^2} + \frac{4}{\sqrt{5}} \frac{dx}{dt} + 8x = 0$$

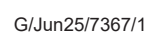
[5 marks]

[illegible]

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[6 marks]

[illegible]

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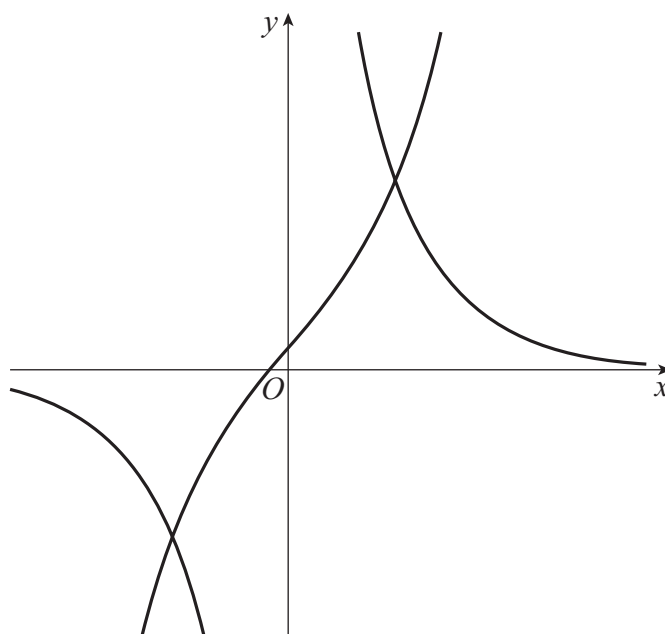
[illegible]

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- 18** The diagram shows part of the graph of $y = 15 \operatorname{cosech} x$ and part of the graph of $y = 4 \sinh x + \frac{1}{2}$



- 18 (a)** Solve the inequality

$$15 \operatorname{cosech} x < 4 \sinh x + \frac{1}{2}$$

Give your answer in logarithmic form.

[4 marks]



Show that

[4 marks]

[illegible]

Turn over ►



A graph in the xy -plane. The x -axis and y -axis are shown, with the origin labeled O . A parabola opens upwards with its vertex at O . A hyperbola with two branches is also shown, one in the first quadrant and one in the third quadrant. The region R is shaded and is bounded by the parabola, the hyperbola, and the x -axis. The region R is located in the first quadrant, bounded by the x -axis, the parabola, and the hyperbola.

Give your answer in the form $\frac{p}{q} + \ln r + s \ln \left(\frac{t}{3} \right)$ where p, q, r, s and t are integers. [7 marks]

[illegible]

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